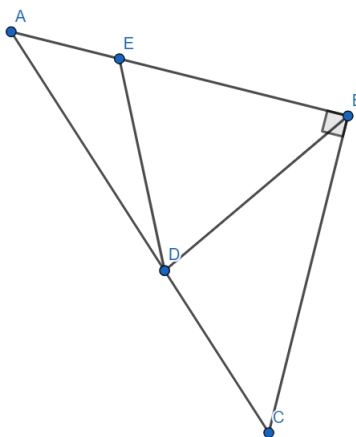


INTEGIRLS Houston: 2022 Fall Math Contest

High School Individual Round

1. I have a deck of cards numbered from 1 through 50. The probability of picking two cards that sum to 99 can be expressed as common fraction $\frac{a}{b}$. What is $a + b$?
2. Engineers at a company are building a glass case for a cube. The glass case must be 1 centimeter thick, and the cube is 6 centimeters long on all sides. How much, in cubic centimeters, glass do the engineers need to build the case?
3. Dave has some money in his wallet. On five separate occasions, he spends $\frac{1}{3}$ of his money. If he ends up with 32, how much money did he spend in total?
4. A turtle travels at a speed of 1 m/s towards a tree. A hare travels in the opposite direction towards the same tree at a speed of 5 m/s. If the turtle is 20 m away from the tree, and the turtle and the hare will reach the tree at the same time, how far are the turtle and the hare from each other?
5. What is the area of the triangle enclosed by the lines $y = 4|x - 1|$ and $y = 2x + 4$?
6. In honor of Pi Day, Celeste bakes a cherry pie. She gives $\frac{1}{4}$ of the pie to her algebra teacher, $\frac{1}{3}$ of the remaining pie to her dog, and $\frac{1}{7}$ of the original pie to herself. If she gives the remainder of the pie to her neighbor and considering that this amount can be represented as $\frac{a}{b}$ in simplest form, what is $a + b$?
7. The polynomial $x^4 + 10x^3 + 35x^2 + 50x + 24$ can be expressed in partially factored form as $(x^2 + 3x + 2)(x + a)(x + b)$. What is $a + b$?
8. Considering that $abcd_4$ is a number in base 4 such that it equals 157_{10} , what is $dcba_4$ in base 10?
9. In right triangle ABC as shown, $AB = BC$. If $BC = 5$, $CD = 2\sqrt{2}$, and $BE = 4$, find $(ED)^2$.



10. A number is “gucci” if it has three or more distinct prime factors. For example, 132 is gucci because its prime factors are 2, 3, and 11. How many positive integers less than 100 are gucci?
11. The first four terms of an arithmetic sequence are x , 27, $3x - 2y$, and $3x + 2y$. What is the 200th term of this sequence?
12. Jose, Miguel, Pedro, and Evelyn want to share a pack of gum with 18 pieces. In how many ways can they distribute the gum so that each person gets at least 3 pieces and no more than 5 pieces?
13. How many solutions are there to the equation $(4 - x^2)^{x^2-19} = 1$?
14. The floor function $\lfloor x \rfloor$ takes a real number input and outputs the greatest integer less than or equal to x . How many integers n are there such that $0 < n < 100$ and $\lfloor \sqrt{n} \rfloor$ is a prime number?
15. Sandra wakes up early in the morning, so early that it’s usually dark outside, and she can’t see the colors of the socks in her sock drawer. Sandra’s sock drawer has 10 blue socks, 6 red socks, and 20 pink socks with rabbits. She hates pink socks with rabbits. How many socks does she have to take out to be certain that she has a matching pair of socks that are not a pair of pink socks with rabbits?
16. What is the remainder when $22!$ is divided by 23?
17. ABC is a right isosceles triangle on the coordinate plane. Angle ABC is right, and B lies at the origin. The coordinates of A and C are $(a, 5)$ and $(b, 12)$ respectively, for real numbers a and b . What is twice the area of ABC ?
18. Find for what value positive integer k the following equation holds true:

$$\left(\sum_{n=1}^k \log_5 n^2 \right)^2 + \left(\sum_{n=1}^k \log_5 n^3 \right)^2 = 13(1 + \log_5 24)^2$$

19. Let $x^3 - ax^2 - 41x + 42$ be a polynomial with real roots such that the sum of two of them is equal to the reciprocal of the third, and a is a real number. What is a ?
20. Maya rolls a fair dice n times. The probability she that rolls the number 6 exactly two times is 5 times as much as the probability that she rolls the number 6 exactly 3 times. What is n ?