INTEGIRLS Houston: 2022 Fall Math Contest

High School Team Round

1. What is the least six-digit palindrome that is divisible by 12?
2. A positive integer is a 4-tripleprime if it can be expressed as a product of three numbers n, $n+4$, $n+6$. Calculate the least 3-digit 4-tripleprime.
3. Enya rolls one fair, six-sided die. She then calculates the area of a square with sides of length of the number she rolled. What is the value of the number that is six times the expected value of the area of the square?
4. If the side length of a regular pentagon is 1, the length of one of its diagonals can be expressed in simplest radical form as $\frac{a+\sqrt{b}}{2}$. Compute $a+b$.
5. There exists N six-digit positive integers such that the product of their digits is 1000. Compute N.
6. Compute the least real solution of $r^3+3r^2+3r+1=r+1$.
7. Cookies are arranged on a hexagonal tray. If a maximum of seven perfectly circular cookies fit on the tray, and the tray is a regular hexagon with area $8\sqrt{3}+12$, find the diameter of each cookie.
8. The area of a regular octagon with side length $\sqrt{2}$ can be expressed as $a+b\sqrt{c}$. Find $a+b+c$.
9. Claire chooses a four-digit number with four distinct nonzero digits. She then obtains the 24 permutations of the number's digits and finds that exactly 12 of them are greater than 5000. What is the least possible value of Claire's initial number?
10. When converted into decimal form, $\frac{A}{B}$ and $\frac{B}{C}$ are both positive repeating decimals (possibly greater than 1). A, B, and C are all positive integers. If $A+B+C=100$, what is the greatest possible value of $B-C$?