INTEGIRLS Houston: 2022 Fall Math Contest

Middle School Team Round

1. If the second and sixth terms of a geometric sequence are 18 and 1458, respectively, what is the 103rd term divided by the 100th term?

2. Simone has 6 sticks of equal length. She uses 3 of the sticks to form an equilateral triangle with area 20. Then, Simone uses all 6 sticks to form a regular hexagon with area A. What is A?

3. Circle A's radius divided by circle B's diameter is 4. What is the area of circle A divided by the area of circle B?

4. Jermaine has 15 oranges. He gives some positive number of oranges to Katie, gives some positive number of oranges to Lucy, and is left with the average of how much he gave to each of them for himself. In how many ways can this be done?

5. Line segment MN is tangent to a circle with center O at point N. $MO = 7$ and $MN = 5$. What is the square of the diameter of circle O?

6. Mia buys a big bouquet of red and yellow roses. From that big bouquet he makes a small bouquet of roses, using $\frac{1}{3}$ of the yellow roses in the big bouquet and $\frac{1}{4}$ the red roses in the big bouquet. The small bouquet has 10 flowers. Given that there were originally more red roses than yellow roses in the big bouquet, what is the least possible number of red roses originally in the big bouquet?

7. The sum $\frac{1}{4} + \frac{2}{16} + \frac{3}{64} + \frac{4}{256}$ can be expressed as a fraction $\frac{a}{b}$, where a and b are positive and the greatest common factor between a and b is 1. What is $a + b$?

8. Diameter MN is drawn in a circle. Chord PQ is drawn parallel to MN. If MN and PQ are 48 units apart and $MN = 100$, then what is PQ?

9. $ABCD$ is a trapezoid with right angles at B and C. $AB = 5$ and $BC = 4$. A circle can be inscribed in $ABCD$ so that it is tangent to each side of $ABCD$ at exactly one point. The area of $ABCD$ can be expressed as a common fraction $\frac{a}{b}$. What is $a + b$?

10. Let $P(x)$ be a polynomial of degree 3 such that $P(1) = 2$, $P(2) = 4$, $P(3) = 8$, and $P(4) = 16$. Find $P(5)$.