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Ductile-Regime Grinding: A New Technology for Machining Brittle Materials

Because of recent advances in precision engineering that allow controlled grinding infeed rates as small as several nanometers per grinding wheel revolution, it is possible to grind brittle materials so that the predominant material-removal mechanism is plastic-flow and not fracture. This process is known as ductile-regime grinding. When brittle materials are ground through a process of plastic deformation, surface finishes similar to those achieved in polishing or lapping are produced. Unlike polishing or lapping, however, grinding is a deterministic process, permitting finely controlled contour accuracy and complex shapes. In this paper, the development of a research apparatus capable of ductile-regime grinding is described. Furthermore, an analytical and experimental investigation of the infeed rates necessary for ductile-regime grinding of brittle materials is presented. Finally, a model is proposed, relating the grinding infeed rate necessary for ductile material-removal with the properties of the brittle workpiece material.

Introduction

When machined, a brittle material can deform through a variety of mechanisms. If the critical resolved shear stress at any point within the material exceeds the elastic yield stress, the mechanism of deformation will change from one of reversible energy storage via elastic stretching to one of irreversible energy dissipation. Examples of irreversible deformation include macroscopic fracture propagation, microcrack formation, phase transformation, dislocation motion (in crystals), and intermolecular sliding (in amorphous materials). Irreversible material-removal mechanisms can be divided into two types: brittle and ductile. In brittle mechanisms, material removal is accomplished through the propagation and intersection of cracks, while ductile mechanisms produce plastic flow of material in the form of severely sheared machining chips.

Recent improvements in machining tolerances have exposed a new possibility for material-removal from brittle substances. It has been noted that plastically deformed chips are formed in the machining of brittle substances if the scale of the machining operation is small (less than 1 μm depth of cut) (Toh McPherson, 1986). Similar ductile chip-formation has been observed in fine scale machining debris from a wide range of ceramics, glasses, and crystals (Huerta and Malkin, 1976; Bifano et al, 1987; Yoshioka et al, 1984; Molloy et al, 1987). This suggests that the process of ductile chip formation may be independent of nature of the material (e.g, brittle or ductile, hard, or soft, crystalline or amorphous, etc.). Grinding of brittle materials under conditions that allow predominantly ductile material-removal is a new technology known as *ductile-regime grinding*, which is the subject of this paper.

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The transition from brittle to ductile material-removal at smaller cutting depths can be argued purely from considerations of material-removal *energy*. Specifically, for lower machining depths-of-cut, it can be shown that plastic flow is a more energetically favorable material-removal process than fracture. The material property characterizing resistance to plastic flow is the yield stress, σ_y. The energy (E_p) required to plastically deform a specified volume of material (V_p) can be written as:

$$E_p = \sigma_y V_p \tag{1}$$

The material property characterizing resistance to fracture is the Griffith crack propagation parameter, G. The energy (E_f) required for fracture is a function of the area (A_f) of new surface created by crack propagation. Thus:

$$E_f = GA_f \tag{2}$$

For a machining depth d, it is reasonable to assume that the order of magnitude of both V_p and a_f are determined by d. That is:

$$V_p \sim d^3 \tag{3}$$

$$A_f \sim d^2 \tag{4}$$

The ratio of material-removal energies, then, is given by

$$\frac{\text{Plastic Flow Energy}}{\text{Fracture Energy}} \sim \frac{E_p}{E_f} \propto d \tag{5}$$

Consequently, as the scale of machining decreases, plastic flow becomes an energetically more favorable material-removal mechanism. The specific depth at which a brittle-ductile transition occurs is a function of the intrinsic material properties governing plastic deformation and fracture.

Consideration of this energy argument leads to a generalization that we named the "Ductile-Regime Grinding Hypothesis." This hypothesis states that for any material, if the dimensional scale of material-removal is made small enough, material-removal will proceed by a mechanism of plastic flow and not fracture. A grinding apparatus capable of achieving this ductile regime of material-removal can be used to combine the fine tolerances that are achievable in a deterministic grinding process with the superfinished surfaces that are achievable in a nondeterministic polishing or lapping process.

In grinding, the "scale" of material-removal is characterized by the depth-of-cut imposed on a given abrasive grit. For plunge grinding, this parameter is determined by the grinding infeed rate. Therefore, the hypothesis implies that in plunge grinding, there will be a critical-grinding-infeed-rate, below which no fracture will occur for a given brittle workpiece material.

One way of viewing the ductile-regime grinding problem is that there is a challenge, first described by Miyashita, to fill a gap in the achievable material-removal rate in abrasive machining (Miyashita, 1985). If the currently achievable material-removal rates for grinding and polishing are compared, there is a gap in which neither technique has successfully been utilized. This region of material-removal has been termed the mircogrinding gap. For grinding processes, material removal is accompanied by localized fracture for virtually all brittle materials (Molly et al, 1987; Huerta and Malkin, 1976; Yoshoika et al, 1982). Polishing processes, on the other hand, result in material removal without fracture. The importance of this material-removal rate gap, then, is that it represents the threshold between ductile and brittle grinding regimes for a wide range of glasses, ceramics, and semiconductors (Bifano, 1988).

This paper describes a research effort to characterize the physical parameters that control the brittle-to-ductile transition in the grinding of brittle materials. The results of this investigation lend strong evidence in support of the ductile-regime grinding hypothesis. Also, a model is proposed, defining the brittle-to-ductile transition in terms of the material properties of the workpiece and the rate of material removal.

In this paper, three aspects of the research are described:

- Development of a grinding apparatus capable of ductile-regime grinding on brittle materials. The relevant design features include:
  - — Machine configuration
  - — Machine Stiffness
  - — Infeed control system
  - — Wheel trueing techniques
- Experimental evaluation of the grinding apparatus, in terms of its capacity for ductile-regime grinding on various materials
- Formulation and evaluation of a ductile-regime grinding model.

The Evolution of Ductile-Regime Grinding

The possibility of grinding brittle materials in a ductile manner was proposed as early as 1954, when it was noted that during frictional wear of rock salts, the dominant material-removal process was plastic flow and not fracture (King and Tabor, 1954). By 1975, improvements in precision diamond grinding mechanisms allowed the first reproducible evidence of grinding ductility in brittle glass workpieces (Huerta and Malkin, 1976). Evidence of the brittle-ductile transition in the grinding of glass appeared as both improvements in surface finish and changes in the specific grinding energy (Chandrasekar and Sathyanarayanan, 1987).

The first systematic studies of grinding ductility were performed using a single grit grinding apparatus. The material-removal regime in these experiments was shown to progress through the three stages: plastic grooving, generation of median and lateral cracks, and finally crushing (Swain, 1979). In this study, it was demonstrated that the progression of material-removal mechanisms was directly related to the force on the abrasive grain, with lower forces corresponding to a decrease in the observed surface fracture. Other experiments of single grit abrasion tests on myriad brittle materials including glasses (Schinker and Doll, 1987; Molloy et al., 1987; Yoshioka et al, 1982), semiconductors (Danyluk, 1986), and advanced ceramics (Swain, 1979; Toh and McPherson, 1986) demonstrated similar transitions in the material-removal process as a function of the grinding force or depth of cut.

The first grinding apparatus specifically designed to take advantage of the relationship between small grinding infeeds (0.2 μm per pass on a surface grinder) and improved surface finish was built by Yoskioka (Yosioka et al., 1982). With this apparatus, ductile-regime grinding was demonstrated for several brittle, crystalline materials. Later improvements in the precision of this grinder were shown to translate directly into improved surface finish on the brittle workpieces (Yoshioka et al., 1985). It was from this research effort that the concept of the microgrinding gap evolved. While the correlation between higher grinding precision and enhanced grinding ductility was qualitatively demonstrated by these Japanese efforts, quantitative relationships between machine parameters, material properties, and grinding ductility have yet to be established.

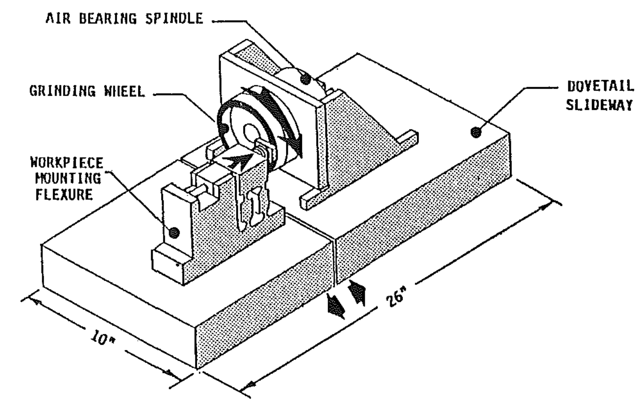


Fig. 1 Schematic of the PEGASUS machine

The Ductile-Regime Grinding Apparatus

Machine Configuration and Actuation Mechanisms. As an initial design criterion, it was decided that a ductile-regime grinder must provide a union of high rigidity (to ensure precision) and low infeed rates (to ensure ductile material-removal). From Yoshioka's early attempts to grind hard materials in the ductile regime, it was established that a grinding infeed resolution of ~50 nm or better is required to prevent significant surface fracture damage in glass (Yoshioka et al, 1982). This represents a level of machine precision that is not often associated with the grinding process. Such motion accuracy, in turn, demands an extremely rigid structural loop so that disturbance forces experienced by either the workpiece or the grinding wheel will not be translated into significant relative motion between the two.

A schematic of the device used in this study is illustrated in Fig. 1. This test bed has been given the acronym PEGASUS (Precision Engineering Grinding Apparatus for Super-finishing Ultrahard Surfaces). The machines provides a mechanism for plunge grinding with the 6 mm wide rim of a 100 mm diameter cup shaped grinding wheel. The workpieces to be ground are rectangular parallelepipeds measuring 6 mm × 6 mm × 18 mm.

Since the purpose of this work is to study the physics of the material-removal processes occurring in microgrinding, it is desirable to limit possible sources of motion error by minimizing the number of degrees of freedom of the system. A plunge grinding apparatus serves well in this regard; in this configuration the only two motions required are rotation of the grinding wheel and infeed of the workpiece. To produce useful components, at least one more axis of motion would be necessary, but all of the essential elements of the material-removal process can be studied using this simplified single-axis plunge geometry. Single axis plunge grinding uses two basic machine motions: rotation of the grinding wheel and infeed of the workpiece. Since ductile-regime grinding demands unusually small relative motion between the workpiece and the grinding wheel, both of these actuation mechanisms must have small (<50 nm) error in motions in the infeed direction. The rotary motion of the grinding wheel is accomplished with a 10 cm diameter air bearing spindle.[1] This spindle allows rotary grinding speeds of up to 5000 RPM with axial error motions less than 40 nm. In addition, all but ~4nm of this axial error in motion is repeatable within a revolution of the spindle and could be accounted for determinsitically. The spindle is rotated by a DC motor having a maximum output torque of 1 Nm. The coupling between the motor and the air bearing spindle consists of two parallel annular flexures, linking the axes of the grinding spindle and the motor.

The infeed mechanism for the PEGASUS apparatus was designed to achieve high rigidity, 2 nm resolution, and relative ease in the fixturing of the workpiece. This workpiece positioning device has three subsystems of increasingly finer precision that work in series in the infeed direction. Work-piece interchange is facilitated by a clamped dovetail slideway with a 15 cm linear range. Preliminary positioning of the workpiece with respect to the grinding wheel face is achieved with a clamped double-reed flexure assembly, actuated by an 80-pitch manually-adjusted lead screw. The parallel flexures ensure a nearly horizontal translation of the workpiece over a 0.5 mm range of travel with 1 μm resolution. A pair of clamping bolts permits rigid fixturing of this assembly before the actual grinding infeed is begun. The final tier of motion actuation along the workpiece infeed axis is that of grinding infeed itself. This is accomplished with a preload piezoelectric actuator capable of linear motion over a 10 μm range with a resolution of 2 nm (using closed loop feedback).

Machine Stiffness. Precision ductile-regime grinding requires high machine stiffness to minimize error motions due to disturbance forces and vibration. The PEGASUS apparatus was designed to obtain an appreciable stiffness. The high precision air bearing spindle is the least rigid element in the structural loop, with a stiffness of 120 MN/m in the direction of the grinding infeed. The measured overall axial stiffness of the grinding contact (in the grinding infeed direction) is 52 MN/m. This compares well with the design stiffness, calculated at 50 MN/m. The measured dynamic stiffness of the grinding contact, for a 25 Hz forcing frequency, increases to 116 MN/m. Such rigidity is common to high precision machining systems, e.g., diamond turning machines.

Measurements of the overall machine stiffness fail to consider the local bond stiffness at each diamond abrasive. Since each abrasive grain is seated in an elastic foundation, this stiffness acts in series with the stiffness of the machine's structural loop. Because this bond stiffness influences the local grinding forces, which are directly responsible for grinding ductility, the rigidity of the abrasive grain support is of some concern. While it would be difficult to measure the local bond stiffness it can be theoretically estimated (Bifano, 1988). The relative local stiffness depends linearly on the elastic modulus of the bond material, and this stiffness is ~15 times larger for bronze bonds than for resin bonds. Macroscopically, the small local stiffnesses of the individual diamonds combine in parallel to create an extremely rigid surface.

[1]Professional Instruments Corp. Blockhead, Model 4B

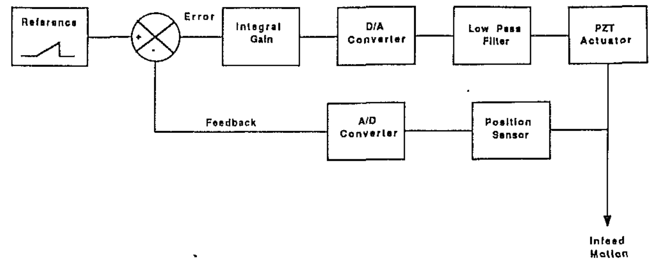


Fig. 2 Schematic of the closed-loop infeed controller

Thus, through selection of a more compliant bond material, the forces of the grinding process can be more evenly distributed to the individual diamond grinding grains, ensuring lower forces on those grains. With lower forces on each individual grain, the cutting depth of each grain will be reduced, resulting in an increased likelihood of ductile material-removal.

Infeed Control System. Perhaps the most important element of the grinding apparatus that determines its capacity for ductile-regime grinding is the regulation of the infeed of the workpiece. Closed-loop real-time feedback control is the only method of ensuring sub-micrometer precision for this infeed, and is thus a necessary component of the grinding apparatus. Based on the relative lack of complexity of the transfer function and low grinding infeed rates required, an integral feedback scheme was chosen as the system controller. This type of control is relatively insensitive to the system model, and ensures an elimination of all steady state errors. In addition, this simple controller has proven quite successful in previous precision actuation systems using similar actuators (Bifano and Dow, 1985; Dow et al., 1989). The control algorithm is shown schematically in Fig. 2. The integral gain and the frequency cutoff of the low-pass filter are the two variables that determine the accuracy and smoothness of the infeed motion. Both were experimentally optimized, and permit infeed rates from 2 nm/rev to 1.25 μm/rev.

Trueing the Grinding Wheel. Trueing is the process of reducing the runout of the mounted grinding wheel so that the contact forces between the abrasive grains and the workpiece can be reliably controlled. Trueing accuracy represents an additional factor that needs to be controlled to ensure knowledge of the position of the workpiece with respect to the grinding wheel. In the PEGASUS configuration, this requires machining the rim of the cup wheel to reduce its axial runout. Since the wheel is mounted on an air bearing spindle that has an unrepeatable runout of less than 4 nm, this value represents a lower limit of the obtainable trueness of the grinding wheel on this apparatus.

The trueing technique that was found to be most successful was that of diamond paste "lapping" of the grinding wheel. Feeding a pyrex lap into the grinding wheel with a generous supply of diamond paste in the contact region proved effective in reducing the runout from 4 μm to less than 0.12 μm peak-to-valley after four 100 μm infeeds of the lap (see Fig. 4). This level of trueing accuracy ranks as the state-of-the-art for diamond grinding wheels.

It was found that a paste composed of diamonds (9 μm diameter) that were about the same size as the diamonds in the wheel (4–8 μm diameter) were most efficient in the trueing process, as compared to larger (25 μm diameter) and smaller (1 μm and 3 μm diameter) diamond pastes. This trueing technique was effective for both resin and bronze bonded grinding wheels.

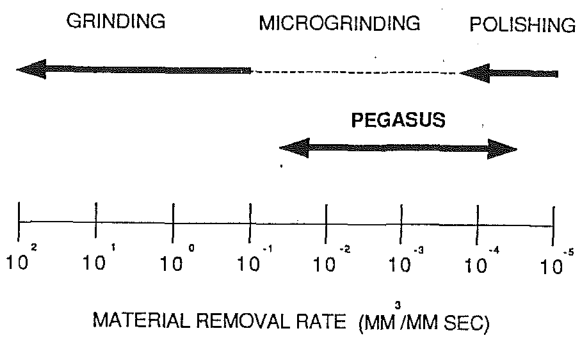


Fig. 3 The "microgrinding gap" versus the operating range of PEGASUS

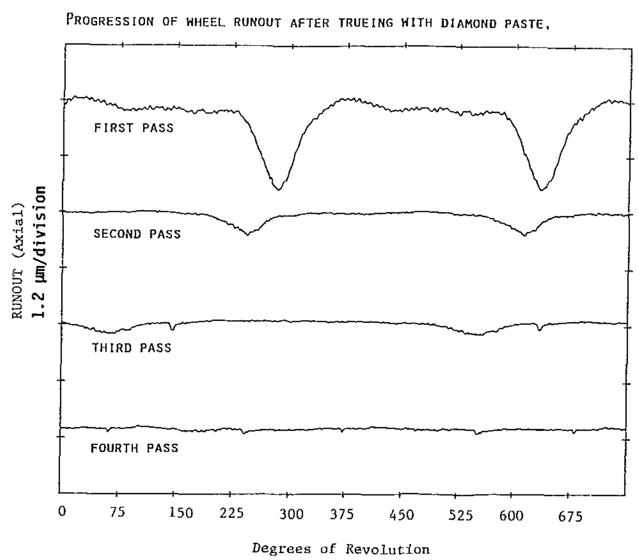


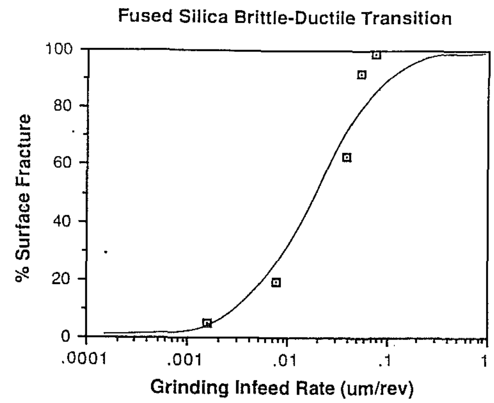
Fig. 4 Reduction in runout after four individual infeeds of 10 μm each, using pyrex lap and 9 μm grit diamond paste

Summary of Microgrinding Apparatus Design Features. As illustrated in Fig. 3, the microgrinding apparatus described in this chapter extends well into the "microgrinding gap" described previously. Due to its piezoelectric infeed system and its plunge-grinding configuration, PEGASUS is capable of grinding with controlled cutting depths as small as 2 nm. This has proven to be the quintessential requirement for ductile-regime grinding of brittle materials.

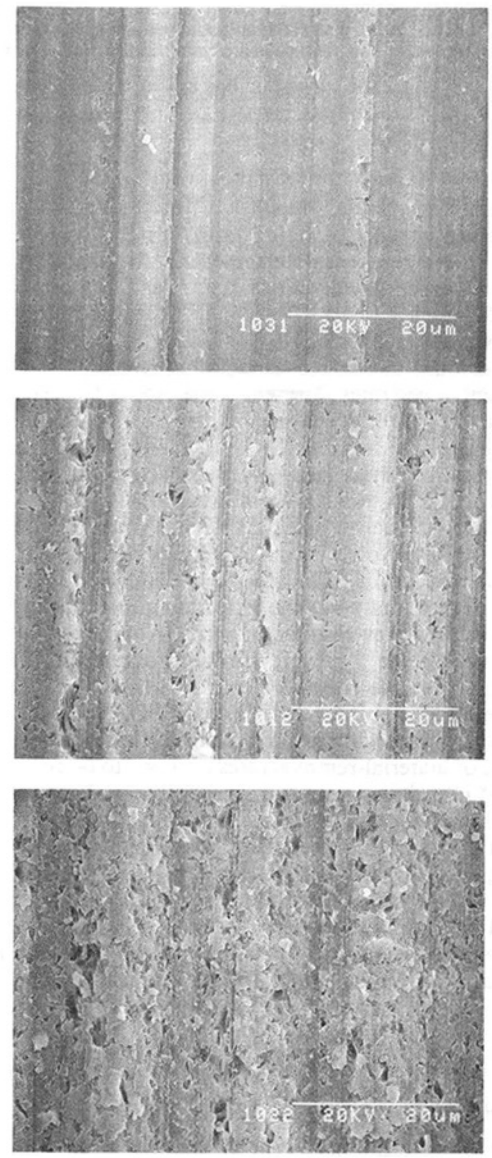
Testing the Ductile-Regime Grinding Hypothesis

Earlier in this paper, a basic hypothesis was postulated for ductile-regime grinding: all materials, regardless of their hardness or brittleness, will undergo a transition from brittle machining regime to a ductile machining regime if the grinding infeed rate is made small enough (Bifano et al., 1987). Below this threshold infeed rate, the energy required to propagate cracks is larger than the energy required for plastic yielding, so plasticity becomes the predominant grinding mechanism. The existence of this transitional infeed rate was demonstrated on the PEGASUS apparatus through a series of test grinds on fused silica.

Using scanning electron microscopy as a post-grinding analysis tool, the ground material surfaces were examined for evidence of surface fracture, which would be indicative of the grinding ductility. A grid counting technique was devised to quantify the real percentage of surface fracture. By applying



(*a*)



(*b*)

Fig. 5 Brittle-ductile transition in fused silica (*a*) Graphical representation of surface fracture versus infeed rate (*b*) Microphotographs corresponding to three different grinding infeed rates. From top to bottom: 2, 7.5, and 37.5 nm/rev.

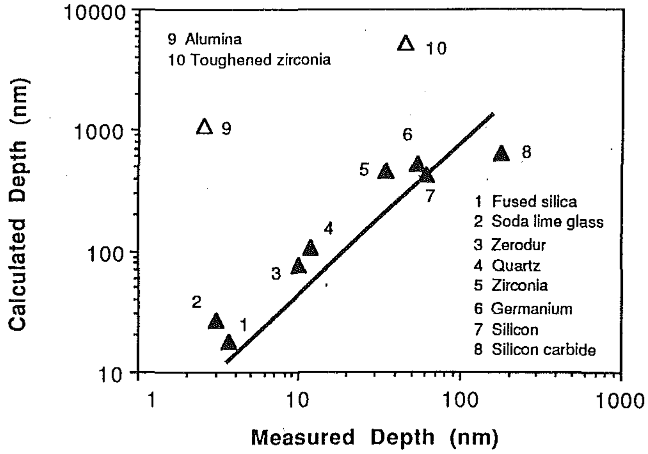


Fig. 6 For each of the 10 materials tested, the measured grinding infeed rate corresponding to the brittle-ductile transition is plotted versus the quantity $\left(\frac{E}{H}\right)\left(\frac{K_c}{H}\right)^2$. The correlation line included has a slope of 1.

this measure of grinding ductility to fused silica samples that were ground using a range of infeed rates, the transitional grinding infeed rate was determined for this material. Figure 5 illustrates this brittle-ductile transition as a function of grinding infeed rate for fused silica. While this grid-counting technique is somewhat subjective, it provides a fair estimate of the relative amount of surface damage between two samples cut at different infeed rates. The technique cannot be used to assess subsurface fracture, which may exist below a smooth, apparently damage-free surface. The subjectivity of the SEM photomicrograph analysis can be reduced by using SEM-based image processing systems, in which the SEM image is digitized and enhanced to define sharp edges and fracture zones. Such a system was used to verify the objectivity of the grid-counting technique that was used in this study (Bifano, 1988).

It was found that a change in grinding infeed rate from 75 nm/rev to 2 nm/rev resulted in a transition from 99 percent surface fracture to 5 percent surface fracture. This result is important for three reasons. First, it verifies that the design principles used in the development of PEGASUS were appropriate for ductile-regime grinding. In addition, the microgrinding range of material-removal rates is shown to be an important region of machining, with significant potential for improving the machinability of glasses and ceramics. Finally, from this series of grinding tests the basic hypothesis of ductile-regime grinding (i.e., brittle-ductile transition for a reduced infeed rate) is validated for fused silica. Fused silica is a relevant material for this verification of the ductile-regime grinding hypothesis because (as will be shown in the next section) it is particularly brittle and thus difficult to machine in a ductile regime.

The Critical-Depth-of-Cut Model

To investigate the influence of material properties on the brittle-ductile transition rate, a broad range of amorphous glasses, single crystals, and advanced ceramics were chosen for grinding on PEGASUS. By comparing the grinding ductility of these materials to their intrinsic properties, a critical-depth-of-cut model has been established for microgrinding. This model relates the measured critical grinding infeed rate for 10 percent surface fracture to a calculated critical-depth-of-cut based on the material properties (10 percent surface fracture was arbitrarily chosen as a reference value for the brittle-to-ductile transition).

The model originates from a formula describing the critical depth for fracture during indentation of hard materials (Lawn, Jensen, and Aurora, 1976). Based on a Griffith fracture propagation criterion, this formula predicts a critical-depth-of-indentation of:

$$d_c = \frac{ER}{H^2} \tag{6}$$

where d_c is the critical indentation depth, E is the elastic modulus, R is the material's fracture energy, and H is the hardness. For materials that exhibit a plastic zone near the crack tip, the value of R can be evaluated using Griffith's classical crack propagation analysis. One approach to defining fracture energy at small scales is to replace it with a dimensionally analogous measure of the energy needed to propagate cracks, namely:

$$R \sim \frac{K_c^2}{H} \tag{7}$$

In indentation, the quantity K_c^2/H has been called an effective measure of brittleness (Marshall and Lawn, 1986). This quantity can be combined with the critical depth model of equation (6) to yield:

$$d_c \propto \left(\frac{E}{H}\right)\left(\frac{K_c}{H}\right)^2 \tag{8}$$

as a measure of the brittle transition depth-of-cut. Experimental results using this formula for indentation testing have shown a remarkable degree of consistency, even using bulk material properties (Marshall and Lawn, 1986). If this type of analysis were adopted for grinding, d_c would represent a measure of the critical-grinding-infeed-rate expected to change the material-removal mechanism from a ductile regime to a brittle one. Thus, presumably a series of brittle materials could be ranked according to their properties to determine the grinding wheel infeed rate below which fracture would not occur.

The relevant properties were measured for each material using microindentation techniques (Bifano, 1988). While the measurement of hardness by indentation is a standard procedure, determination of K_c and E by indentation is a developing area of research (Marshall and Lawn, 1986). The properties of the material surface vary with the indentation depth at which they are measured. This surface property variability is especially troubling for the measurement of K_c. Size-scale effects lead to a dependence of K_c on crack size (R-curve behavior), which can be a large effect in certain materials (Scattergood et al, 1988). Such material-related property variations complicate the extrapolation of properties from the scale of indentation ($\sim$ 10 μm) to the scale of microgrinding (<1 μm). In spite of these problems calculating the material properties, a reasonable correlation was obtained between the calculated critical-depth-of-cut and the measured critical-grinding-infeed-rate (i.e., the grinding infeed that will produce 10 percent surface fracture). This correlation is illustrated by the graph of Fig. 6. From this correlation, the constant of proportionality for eq. (8) can be estimated, yielding:

$$d_c = 0.15 \left(\frac{E}{H}\right)\left(\frac{K_c}{H}\right)^2 \tag{9}$$

Only materials exhibiting significant variations in K_c with indentation depth were not well represented by the model. For these materials, estimates of the K_c behavior at small indentations can be used to modify the calculated critical-grinding-infeed-rate (Bifano, 1988).

Summary

The following is a summary of the results brought out in this paper.

- Ductile-regime grinding is a newly established material-removal technique. By controlling a stiff, accurate grinding apparatus so that it has an exceptionally small scale of material removal, brittle materials can be ground in a ductile manner. As a result, brittle workpieces can be machined in a deterministic process while producing surface finishes characteristic of those achieved in nondeterministic, inherently ductile processes such as lapping and polishing.
- Ductile-regime grinding can be achieved by ensuring that the grinding apparatus has a stiff structural loop, real-time control of the grinding infeed, relative isolation from environmental disturbances, and state-of-the-art wheel trueing techniques.
- All brittle materials will undergo plastic flow rather than fracture if the depth of machining is small enough.
- There is a correlation between the grinding infeed rate that corresponds to the brittle-to-ductile transition for a particular brittle material and the properties (K_c, H, and E) of the material. This correlation is reasonably described by a simple power-law equation.

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