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SUBJECT: AE 2200 Report for Lab 10 – Rocket Launch Analysis

1. Introduction

The purpose of this lab is to compare two rocket engines' /launch vehicles' performances in MATLAB and understand some of the factors considered when selecting an optimal rocket.

2. Background Theory

Calculations were done to find the exit velocity, thrust and specific impulse at different altitudes, the mass ratios of each rocket, and the total initial mass of each rocket for different numbers of stages.

Table 1 below shows the constants used in all further calculations.

Table 1: Provided constants used for results calculations

Constants for Rocket Engine Analysis		
Value	Engine 1	Engine 2
g_0 (Gravitational Constant)	$6.67 \ * \ 10^{-11} \frac{m^3}{kg\,s^2}$	$6.67 \ * \ 10^{-11} \frac{m^3}{kg\,s^2}$
T_0 (Chamber Temperature)	3517 K	3114 K
P_0 (Chamber Pressure)	$20.64 * 10^6\,Pa$	$7.0 * 10^6\,Pa$
d_{exit} (Exit Diameter)	2.30 m	3.70 m
A*/A_e (Throat to Exit Area Ratio	1/69	1/16
P_{exit} (Exit Pressure)	22000 Pa	44500 Pa

γ (Ratio of Specific Heats)	1.22	1.24
Average Molecular Weight	$16 \frac{kg}{kg\,mol}$	$22 \frac{kg}{kg\,mol}$
ΔV (Total Impulse)	9500 m/s	9500 m/s
M_{PL} (Payload Mass)	1000 kg	1000 kg

Equation 1 below is for the mass flow rate ṁ of the respective rocket fuel, which was used to find exit velocity V_{exit} in Equation 2.

$$\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{(\gamma+1)}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}} \tag{1}$$

$$V_{exit} = \sqrt{\frac{2\gamma R T_0}{\gamma-1} \left(1 - \left(\frac{P_e}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}\right)} \tag{2}$$

Where P_0 is the chamber pressure, $A\,*$ is the area of the throat found using the formula for the area of a circle to find exit area and multiplied by the throat to exit area ratio given, T_0 is the chamber temperature, γ is the ratio of specific heats, R is the universal gas constant multiplied by the ratio of the throat to exit areas, and P_e is the exit pressure.

These two values were then plugged into Equation 3 below to solve for the thrust T of the rocket engines.

$$T = \dot{m} V_{exit} + (P_e - P_\infty) A_e \tag{3}$$

Where P_∞ is the ambient pressure and A_e is the exit area.

Mass flow rate and thrust were also used to find the specific impulse, shown in Equation 4 below.

$$I_{SP} = \frac{T}{\dot{m}\, g_0} \tag{4}$$

Where g_0 is the gravitational constant with respect to earth.

The mass ratios for the rocket engines were found using equation 5 below.

$$MR = e^{-\frac{\Delta V}{V_{exit}}} \tag{5}$$

Where ΔV is the given impulse required to achieve low Earth orbit.

Finally, the total initial mass for each rocket for a different number of stages was found using Equation 6 below.

$$M_0 = \frac{M_{PL}}{(e^{-\frac{\Delta V}{V_{exit}}} - \delta)^n} \tag{5}$$

Where M_{PL} is the payload mass, n is the number of stages, and δ is the inert mass ratio.

3. Results

The data sheet with the results of the lab can be found as a separate attached document. The graph figures and values not found in the spreadsheet will be in this lab report.

Table 2 below shows the calculated values for mass flow rate and mass ratio using Equations 1, 2, and 5 respectively.

Table 2: Calculated values for the mass flow rate and mass ratio for rocket engines 1 and 2.

Mass Flow Rate and Mass Ratio for Rocket Engines 1 and 2		
Value	Engine 1	Engine 2
Mass Flow Rate	6.114203020981145E3 m/s	2.461908188287855E04 m/s
Mass Ratio for Single Stage	12.257891334904352	30.918808144006523

The following two figures below, Figure 1 and Figure 2, show the relationship between the number of stages and the initial mass for different given inert mass ratios using Equation 5.

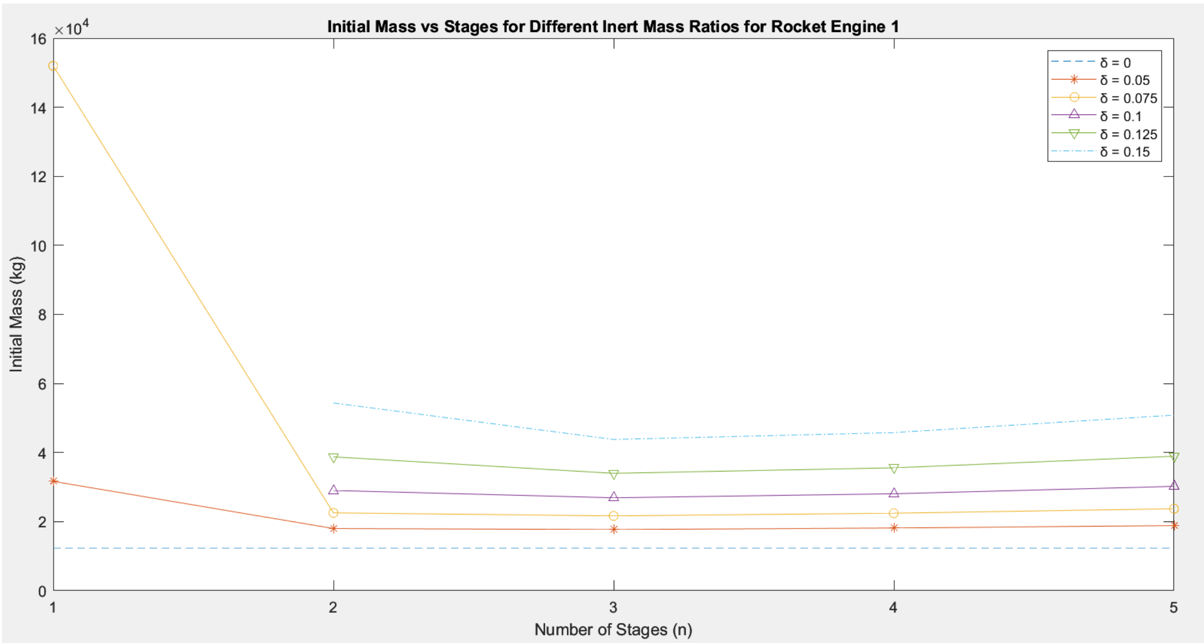

Figure 1: Graph of the initial mass vs number of stages for 6 inert mass ratios for engine 1.

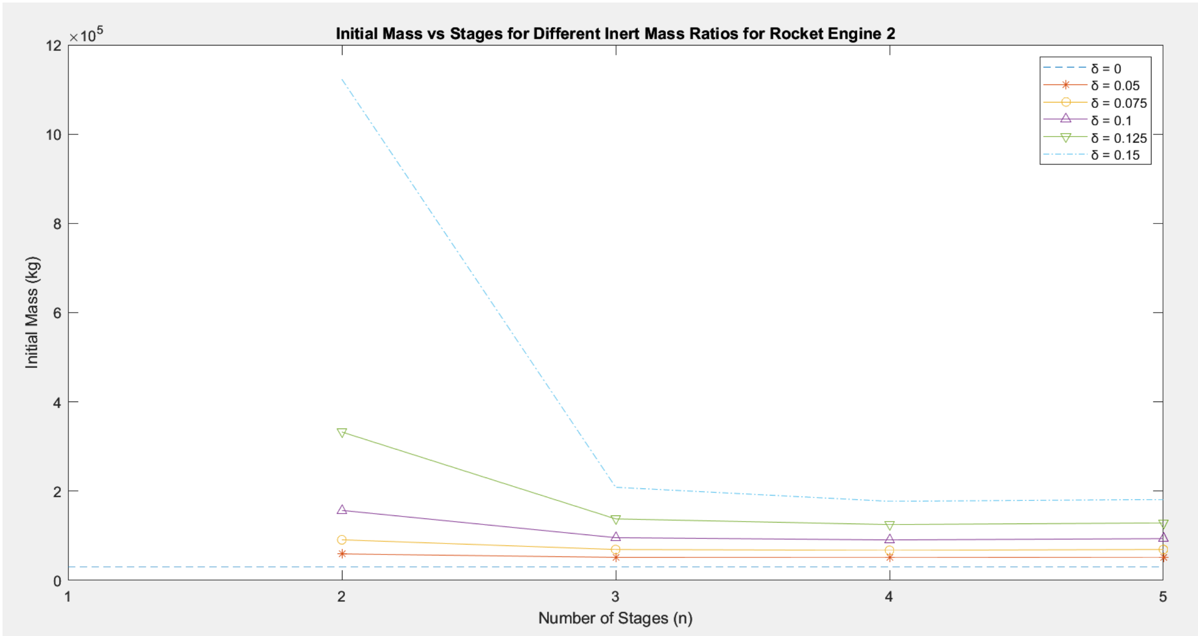

Figure 2: Graph of the initial mass vs number of stages for 6 inert mass ratios for engine 2.

For the two graphs, initial mass values for the first stages of some of the inert mass ratios are not shown. This is due to those values being negative in magnitude, which is realistically impossible as mass cannot be a negative quantity. Therefore, they were not included in the graphs.

4. Discussion / Questions

The design altitudes of each engine were found using the standard atmosphere tables and plugging in the value for exit pressure (exit pressure = ambient pressure). The design altitudes are approximately 11.2 km and 6.4 km for engines 1 and 2 respectively. The design altitudes were likely chosen by the designers so they could have the most optimal ambient pressure for the rocket engine exit velocity since exit pressure impacts exit velocity and ambient pressure equals exit pressure (question 1).

Comparing the engines, engine 2 has a thrust that is around 2.95, or basically 3, times greater than that of engine 1 for all three altitudes. To match the total thrust of engine 2, there would need to be at least 3 engine 1's to match engine 2 (question 2).

There are definite pros and cons to having either many small engines or one large engine. A pro of multiple engines is that it's likely cheaper to make a bunch of smaller engines than one large one, and if one engine were to fail in a launch, then the other engines with it can compensate and the launch may still be possible. If a large engine fails, there is no way the launch can continue. On the other hand, by having more engines, there are more moving parts to keep track of, and more potential sources of error; the more complex you make a system, the more likely an error can arise. A single engine, however, would be more simplistic and have less things in general that could go wrong (question 3).

Based on the observations made in the simulation, as well as the comparison of big vs small engines, I would choose engine 1, the smaller engine, for the rocket, as by being able to make a bunch of smaller rockets, it will likely cost less to make and allow for a fail safe if one engine does experience problems. In the aerospace industry, it costs billions to launch a rocket, so for the sake of cost of building, and ensuring that investment is not wasted by having a single engine ruin the entire launch, the large amount of small engine 1's seems like a good option.

Other designers may choose engine 2 due to its efficiency, since as previously stated 1 engine 2 is worth 3 engine 1's in terms of thrust force, which would also end up weighing more than just the single engine 2. Having a single engine would mean having to integrate less systems, limiting the number of variables that could go wrong (question 4).

Based on the calculations of the lab, I think 3 or 4 stages would be an optimal number for the launch vehicle as according to the graphs, stage 3 has the lowest total initial masses for engine 1, and 4 has the lowest initial masses for engine 2. As indicated in class, cutting down on the mass of the rocket is crucial to its success, so based on the data, having 3 or 4 stages for the launch vehicle depending on which engine is chosen should have the most optimal performance (question 5).

Overall, the lab shows a portion of the analysis and deliberation that goes into the design and selection of a rocket engine and launch vehicle system. As discussed in the analysis, there is no one clear, best solution of what engine to choose, so it is up to engineers to consider all factors and scenarios and pick the most optimal solution they can that will be efficient and cost effective.

Attachment 1: MATLAB Code

```
clc

clear

% Calculate the mass flow rate and exit velocity for each
engine.

% m* = Po*A*/sqrt(T0) * sqrt((y/R)(2/(y+1))^((y+1)/(y-1)))

% ve = sqrt((2*y*R*T0)/(y-1) * (1-(Pe/P0)^((y-1)/y))

T01 = 3517;

P01 = 20.64*10^6;

AE1 = (pi()*(2.3/2)^2);

AT1 = AE1/69;

PE1 = 22000;

y1 = 1.22;

R1 = 8314/16;

MFR1 = P01*AT1/sqrt(T01) *
sqrt((y1/5)*(2/(y1+1))^((y1+1)/(y1-1)));

VE1 = sqrt((2*y1*R1*T01)/(y1-1) * (1-(PE1/P01)^((y1-1)/y1)));

T02 = 3144;

P02 = 7*10^6;

AE2 = (pi()*(3.7/2)^2);

AT2 = AE2/16;

PE2 = 44500;

y2 = 1.24;

R2 = 8314/22;
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```
MFR2 = P02*AT2/sqrt(T02) *
sqrt((y2/5)*(2/(y2+1))^((y2+1)/(y2-1)));
VE2 = sqrt((2*y2*R2*T02)/(y2-1) * (1-(PE2/P02)^((y2-1)/y2)));
% Calculate the thrust and specific impulse for each engine at
sea level,
% at its design altitude (where the flow is ideally expanded),
and in a vacuum.
% T = m*Ve + (Pe-P0)*A*
g0 = 6.67 * 10^-11;
PSLV = 101325;
TSLV1 = MFR1*VE1 + (PE1 - PSLV)*AE1;
ISPSLV1 = TSLV1 / (MFR1 * g0);
TDA1 = MFR1*VE1;
ISPDA1 = TDA1 / (MFR1 * g0);
TVC1 = MFR1*VE1 + (PE1)*AE1;
ISPVC1 = TVC1 / (MFR1 * g0);
TSLV2 = MFR2*VE2 + (PE2 - PSLV)*AE2;
ISPSLV2 = TSLV2 / (MFR2 * g0);
TDA2 = MFR2*VE2;
ISPDA2 = TDA2 / (MFR2 * g0);
TVC2 = MFR2*VE2 + (PE2)*AE2;
ISPVC2 = TVC2 / (MFR2 * g0);
% Rocket Launch Analysis
DV = 9500;
```

```
% Calculate the required MR for a single stage rocket to put a
payload into
% orbit for each of the two engine options.
% DV = -Veln(MR)
MR11 = exp(DV/VE1);
MR12 = exp(DV/VE2);
% Calculate the total initial mass of a single stage to orbit
(SSTO) rocket
% to launch a 1000 kg payload into orbit for each of the two
engine options,
% for inert mass ratios of 0, 0.05, 0.075, 0.1, 0.125 and 0.15.
If it is
% impossible for a given scenario, indicate that with N/A
instead of giving a mass.
% Inert mass ratio = MIN/M0
% Payload mass ratio = MPL/M0
% (MPL + MIN)/M0 = PMR + IMiR
% Calculate the total initial mass of a two stage rocket to
launch a 1000 kg
% payload into orbit assuming each stage contributes the same
to the rocket.
% Do this for each of the two engine options, and for inert mass
ratios of
```

```
% 0, 0.05, 0.075, 0.1, 0.125 and 0.15.  If it is impossible for
a given
% scenario, indicate that with N/A instead of giving a mass.
% M0 = MPL / e ^ DV/nVe - d
% Calculate the total initial mass of a multistage rocket to
launch a 1000
% kg payload into orbit assuming each stage contributes the same
to the
% rocket, for n = 3, 4 and 5 stages.  Do this for each of the
two engine
% options, and for inert mass ratios of 0, 0.05, 0.075, 0.1,
0.125 and 0.15.
% If it is impossible for a given scenario, indicate that with
N/A instead
% of giving a mass.
% M0 = MPL / e^-(DV/nVe) - MINR
MPL = 1000;
MINR = [0,0.05,0.075,0.1,0.125,0.15];
for j =1:1:6
   M0S11(j) = MPL / (exp(-DV/(VE1))-MINR(j));
   M0S21(j) = MPL / ((exp(-DV/(2*VE1))-MINR(j)))^2;
   M0S31(j) = MPL / ((exp(-DV/(3*VE1))-MINR(j)))^3;
   M0S41(j) = MPL / ((exp(-DV/(4*VE1))-MINR(j)))^4;
   M0S51(j) = MPL / ((exp(-DV/(5*VE1))-MINR(j)))^5;
```

```
   M0S12(j) = MPL / (exp(-DV/(VE2))-MINR(j));
   M0S22(j) = MPL / ((exp(-DV/(2*VE2))-MINR(j)))^2;
   M0S32(j) = MPL / ((exp(-DV/(3*VE2))-MINR(j)))^3;
   M0S42(j) = MPL / ((exp(-DV/(4*VE2))-MINR(j)))^4;
   M0S52(j) = MPL / ((exp(-DV/(5*VE2))-MINR(j)))^5;
end
% Create two figures (one for each engine) comparing M0 for each
number of
% stages (1-5) with each inert mass ratio as a separate data
curve.
n = (1:5);
M0MR11 = [M0S11(1),M0S21(1),M0S31(1),M0S41(1),M0S51(1)];
M0MR21 = [M0S11(2),M0S21(2),M0S31(2),M0S41(2),M0S51(2)];
M0MR31 = [M0S11(3),M0S21(3),M0S31(3),M0S41(3),M0S51(3)];
M0MR41 = [M0S11(4),M0S21(4),M0S31(4),M0S41(4),M0S51(4)];
M0MR51 = [M0S11(5),M0S21(5),M0S31(5),M0S41(5),M0S51(5)];
M0MR61 = [M0S11(6),M0S21(6),M0S31(6),M0S41(6),M0S51(6)];
M0MR12 = [M0S12(1),M0S22(1),M0S32(1),M0S42(1),M0S52(1)];
M0MR22 = [M0S12(2),M0S22(2),M0S32(2),M0S42(2),M0S52(2)];
M0MR32 = [M0S12(3),M0S22(3),M0S32(3),M0S42(3),M0S52(3)];
M0MR42 = [M0S12(4),M0S22(4),M0S32(4),M0S42(4),M0S52(4)];
M0MR52 = [M0S12(5),M0S22(5),M0S32(5),M0S42(5),M0S52(5)];
M0MR62 = [M0S12(6),M0S22(6),M0S32(6),M0S42(6),M0S52(6)];
plot(n,M0MR11,'--')
```

```
hold on

plot(n,M0MR21,'-*')

hold on

plot(n,M0MR31,'-o')

hold on

plot(n(2:5),M0MR41(2:5),'-^')

hold on

plot(n(2:5),M0MR51(2:5),'-v')

hold on

plot (n(2:5),M0MR61(2:5),'-.')

xticks([1 2 3 4 5]);

title('Initial Mass vs Stages for Different Inert Mass Ratios
for Rocket Engine 1')

xlabel('Number of Stages (n)')

ylabel ('Initial Mass (kg)')

legend ('δ = 0','δ = 0.05','δ = 0.075','δ = 0.1','δ = 0.125','δ
= 0.15')

figure

plot(n,M0MR12,'--')

hold on

plot(n(2:5),M0MR22(2:5),'-*')

hold on

plot(n(2:5),M0MR32(2:5),'-o')

hold on
```

```
plot(n(2:5),M0MR42(2:5),'-^')

hold on

plot(n(2:5),M0MR52(2:5),'-v')

hold on

plot(n(2:5),M0MR62(2:5),'-.')

xticks([1 2 3 4 5]);

title('Initial Mass vs Stages for Different Inert Mass Ratios
for Rocket Engine 2')

xlabel('Number of Stages (n)')

ylabel ('Initial Mass (kg)')

legend ('δ = 0','δ = 0.05','δ = 0.075','δ = 0.1','δ = 0.125','δ
= 0.15')
```