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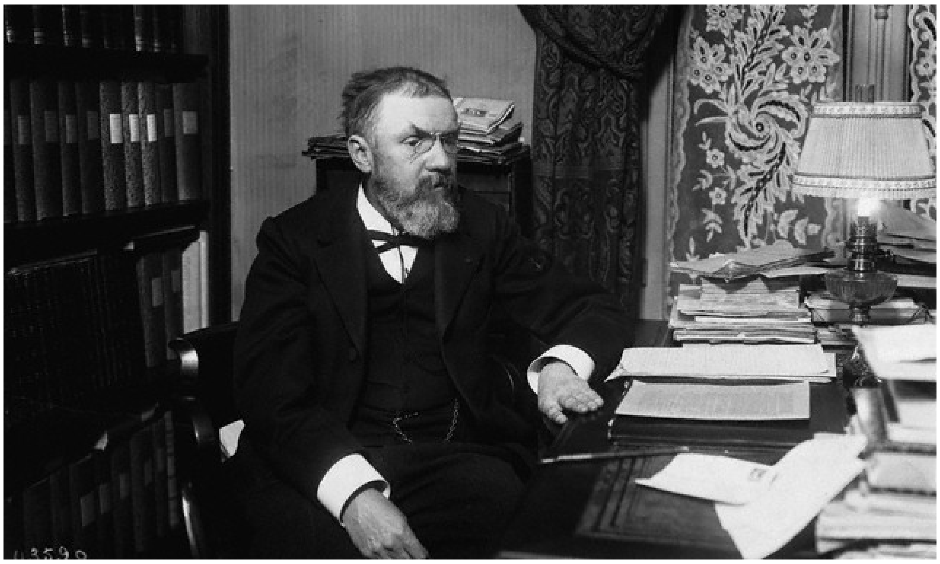
An interview with Henri Poincaré

Mathematics is the art of giving the same name to different things

It looked like a daunting and perhaps even impossible enterprise to interview the famous professor Henri Poincaré. However, it turned out to be possible on the condition that professor Poincaré was permitted to formulate the questions himself. We were happy to accept this condition. The interview is elaborated by Ferdinand Verhulst.

[We start with some questions about the foundations of mathematical thinking.]
Question: What is the nature of mathematical reasoning? Is it really deductive as is usually believed? [1, essay 'Sur la nature de la raisonnement mathématique']
Answer: A deep analysis shows us that it is not, that it uses to a certain measure inductive reasoning and in this way it is fruitful. Opening an arbitrary book on mathematics, we find the author announcing that he wants to generalize a known proposition. So, the mathematical method proceeds from the particular to the general and how is it then that we can call it deductive?


Henri Poincaré receives visitors in his office at home

[Thinking about the foundations of mathematics can be interesting, but a question for the 'mathematician at work' is whether exploring the foundations actually improves our mathematical reasoning.]
Question: Have we achieved absolute rigour [in mathematics]? In each stage of the evolution, our predecessors believed to have achieved this. If they were wrong, are we not also wrong like them? [2, essay 'L'intuition et la logique en mathématiques']
Answer: We believe that in our reasoning, we don't need intuition. The philosophers tell us that this is an illusion. Pure logic will lead us only to tautologies. This cannot create anything new, from logic alone no science can emerge. For the other thing we need, we have no other word than 'intuition'. If one wants to take the trouble to be rigorous in todays' analysis, there are only syllogisms or an appeal to the intuition of numbers [induction], the on-

ly one which cannot deceive us. One can say that today absolute rigour has been achieved.

[You have been very productive in mathematics and in other fields, so a few questions about research strategy are of interest to us.]

Question: Pure analysis puts a great many procedures at our disposal with a guarantee for correct answers. But, from all the roads we can take, which one will lead us as quickly as possible to the goal? Who will tell us which one to choose? [2, essay 'L'intuition et la logique en mathématiques']

Answer: We need a faculty that shows us the goal from far away and this faculty is 'intuition'. Logic and intuition both play a necessary part. Both are indispensable. Logic can give certainty only and is the instrument of proof, intuition is the instrument of invention.

[What can we say about the relation between mathematics, which is concerned with objects of the mind, and the empirical sciences. Some people from the physical sciences will say "mathematics is just a tool, we use as little of it as possible, most mathematics is too artificial".]

Question: Experience is the only source of truth, only this can give us certainty. But if experience is everything, what place remains for mathematical physics? What has experimental physics to do with such a help which seems useless and perhaps even dangerous? [1, essay 'Les hypothèses en physique']

Answer: The scientist must order; one makes science with facts as one makes a house with stones. A big collection of facts is no more a science than a heap of stones is a house. The efforts of scientists have tended in resolving complex phenomena, arising directly from experiments, into a great many of elementary phenomena. The knowledge of elementary facts enables us to put the problems in equations.

Question: What is the objective value of science? And first, what should be understood by objectivity? [2, essay 'La science et la réalité']

Answer: The first condition of objectivity is, that what is objective must be considered as such by a number of spirits, and consequently must be able to be transmitted from one to the other. This transmission can only be done by 'discourse'. No discourse, no objectivity. A second condition is that scientific phenomena correspond with sensations that are actually tested. The first condition separates reality from a dream, the second one from a novel.

[In mathematics we study objects existing only in our mind, we mentioned this before. When we are aware of this, can we expect that mathematical physics leads to knowledge of reality?]

Question: Does science tell us the real nature of things? Does science tell us the real nature of the relation between things? [2, essay 'La science et la réalité']

Answer: About the first question nobody hesitates to answer 'no'. But I believe one can go further: not only can science not tell us about the nature of things, nothing is able to tell us that and if some god knew it, he could not find the words to express it. Not only could we not guess the answer, but if one would give it to us, we would not be able to understand it. I even ask myself whether we have a good understanding of the question.

To understand the meaning of the second question we have to consider the conditions of objectivity. Are these relations the same for everybody? It is essential that everybody who is knowledgeable on experiments agrees about it. The question is then whether this agreement will persist with our successors. One will say that science is only a classification and that a classification can not be true but only convenient. It is true that it is convenient, but not only for me but for all men. It is true that it will remain convenient for our descendants, it is true finally that this can not be accidental.

[In science, we assume that the physical laws exist for all time and the question of the permanence of physical laws is usually avoided. Can we say something about this?]

Question: Were the laws of nature of former eras those of today? Will the laws of tomorrow still be the same? [4, essay 'L'évolution des lois']

Answer: If we imagine two minds similar to ours observing the universe on two occasions differing for example by millions of years, each of these minds will construct a science which will be a system of laws deduced from observed facts. It is probable that these sciences will be very different and in that sense it could be said that the laws have evolved. But however great the difference may be, it will always be possible to conceive of an intellect, of the same nature as ours but endowed with a much longer life, which will be able to complete the synthesis. To this intellect, the laws will not have changed, science will be unalterable; the scientists will merely have been imperfectly informed.

[Here is a very practical question that has to do with deterministic, but chaotic phenomena in nature. You were the first to write about this in your work on dynamical systems. Roughly 70 years after this, scientists became aware of these fundamental aspects of nature by the papers of Lorenz on an atmospheric model and by Hénon and Heiles on a galactic model. It is amazing that no scientist in physics or engineering picked this up before. The following questions and answers show your insight in the problem of the predictability of real-life phenomena.]

Question: Why have meteorologists such difficulty in predicting the weather with any certainty? [3, essay 'Le hasard']

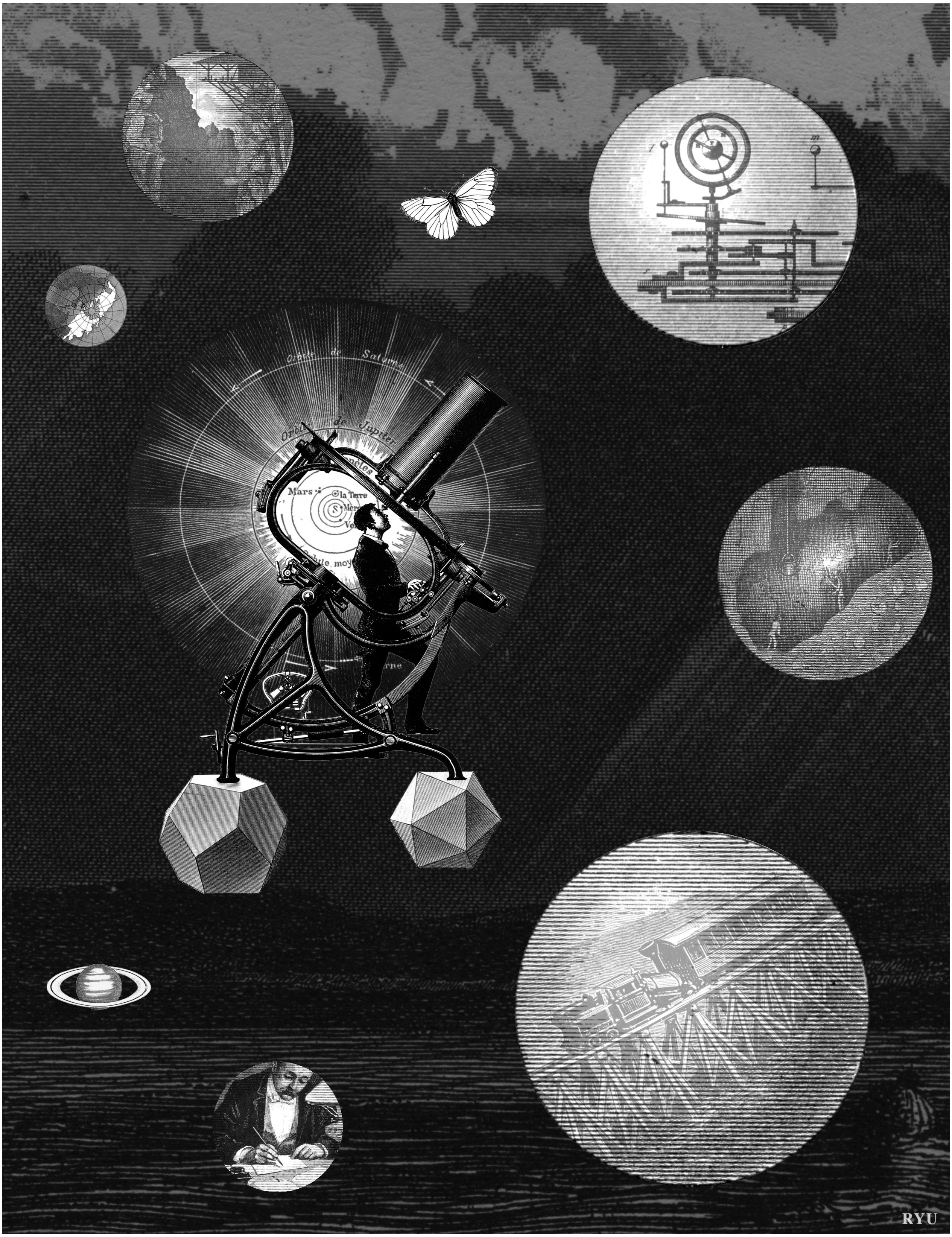
Answer: We see that great disturbances are generally produced in regions where the atmosphere is in unstable equilibrium. The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed somewhere, but they are not in a position to say exactly where. A tenth of a degree more or less at any given point, and the cyclone will burst here and not there, and extends its ravages over districts it would otherwise have spared. Here, again, we find the same contrast between a very trifling cause that is inappreciable to the observer, and considerable effects, that are sometimes terrible disasters.

Question: Do these probability considerations apply outside science? [3, essay 'Le hasard']

Answer: It is the same in the humanities, and particularly in history. The historian is obliged to make a selection of the events in the period he is studying, and he only recounts those that seem to him the most important. Thus he contents himself with relating the most considerable events of the sixteenth century, for instance, and similarly the most remarkable facts of the seventeenth century. If the former are sufficient to explain the latter, we say that these latter conform to the laws of history. But if a great event of the seventeenth century owes its cause to a small fact of the sixteenth century that no history reports and that everyone has neglected, then we say that this event is due to chance, and so the word has the same sense as in the physical sciences; it means that small causes have produced great effects.

The greatest chance is the birth of a great man. No example can give a better understanding of the true character of chance.

[There is a special research topic that touches upon the fundamentals of mathematics, the understanding of the continuum of the real



Illustratie: Ryu Tajiri

numbers. This topic was important in your time but it is still important.]

Question: What is exactly this continuum, this subject of mathematical reasoning? [1, essay ‘La grandeur mathématique et l’expérience’]

Answer: The continuum is nothing else than a set of elements, sequentially arranged, certainly infinite, but separated from each other. Our definition is however not complete. [...] One can ask whether the concept of mathematical continuum has not simply been derived from experience. One is forced to conclude that this idea has been created completely by the human spirit, but that experience has induced it.

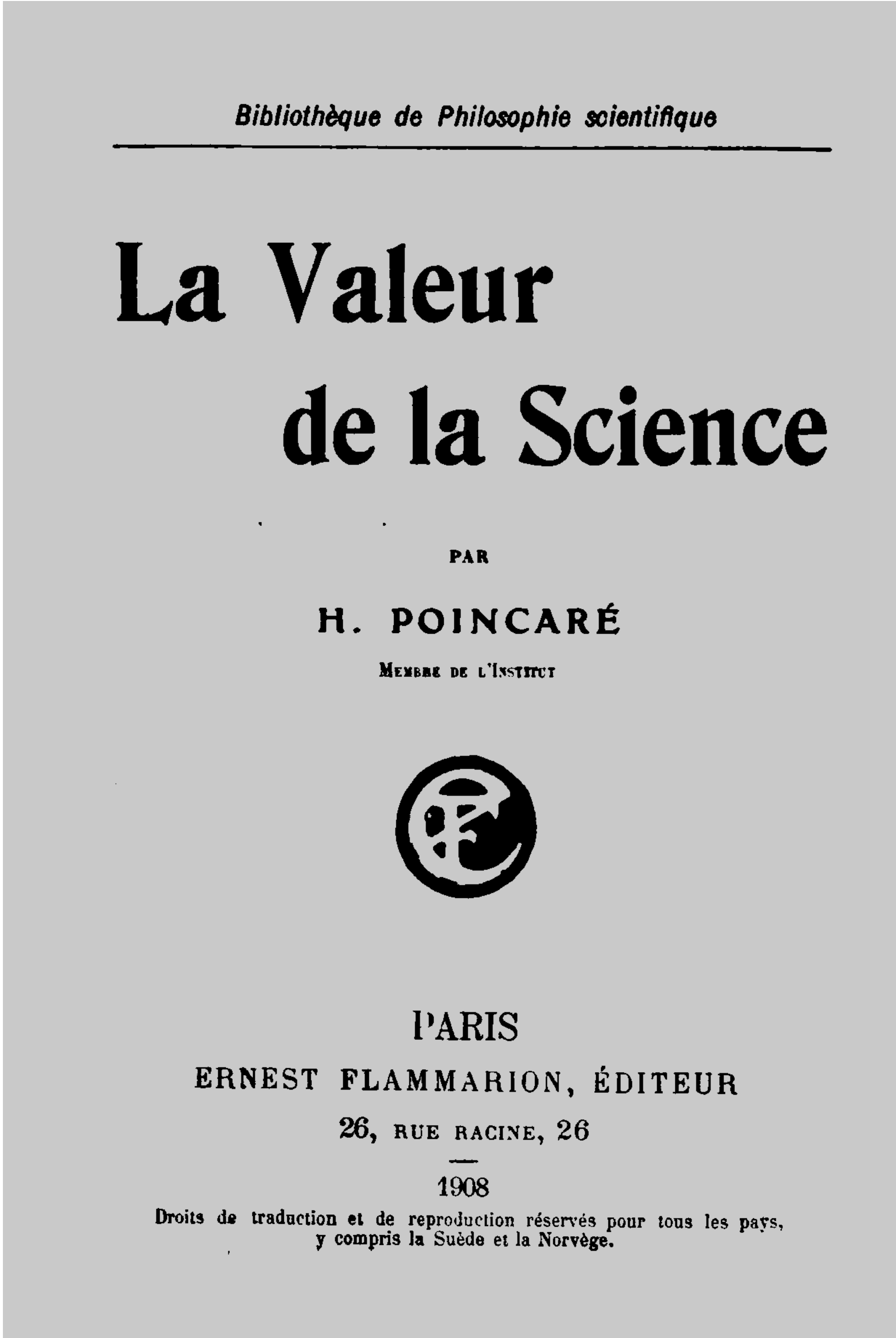
Question: Is the creative power of the mind regarding the mathematical continuum exhausted? [1, essay ‘La grandeur mathématique et l’expérience’]

Answer: No, the work of Du Bois-Reymond shows this in a remarkable way. One knows that mathematicians distinguish between infinitesimal small [quantities] of different orders and that those of second order are not only infinitesimal small in an absolute sense, but also with respect to those of first order. It is not difficult to imagine infinitesimal small quantities of fractional or even irrational order and in this way we find again an ordering of the mathematical continuum.

[The styles and methods of proofs in mathematics have evolved since your time. With the enormous growth of both pure and applied mathematics there is now an abundance of mathematical styles, there are even computer-assisted proofs. Does it make sense to distinguish between beautiful and ugly mathematics, between elegant and graceless reasoning?]

Question: Mathematicians attach a great importance to the elegance of their methods and of their results, and this is not mere dilettantism. What is it that gives us the feeling of elegance in a solution or proof? [3, essay ‘L’avenir des mathématiques’]

Answer: It is the harmony of the different parts, their symmetry, and their happy adjustment; it is, in a word, all that introduces order, all that gives them unity, that enables us to obtain a clear comprehension of the whole as well as of the parts. Elegance may result from the feeling of surprise caused by the unlooked-for occurrence of objects not habitually associated. In this, again, it is fruitful, since it discloses thus relations that were until then unrecognized. Mathematics is the art of giving the same names to different things.



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La Valeur de la Science

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H. POINCARÉ

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PARIS

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[In your time, you were one of the last universal scientists, but then, scientists started to specialize in their research. Is this a danger or a necessity, or both?]

Question: How should one view specialization? [3, essay ‘L’avenir des mathématiques’]

Answer: As science develops, it becomes relatively more difficult to grasp it in its entirety. Then an attempt is made to cut it in pieces and to be satisfied with one of these pieces — in one word, to specialize. Too great a movement in this direction would constitute a serious obstacle to the progress of science. As I have said, it is by unexpected concurrences between its different parts that it can make progress. Too much specialization would prohibit these concurrences.

[You wrote many textbooks, but why are some people afraid of mathematics? To prove and invent new mathematics, one needs intuition, but to understand school mathematics one needs only some natural logic.]

Question: How is it that there are so many minds that are incapable of understanding

[even elementary] mathematics? Here is a science which appeals only to the fundamental principles of logic; is there not something paradoxical in this? [3, essay 'Les définitions mathématiques et l'enseignement']
Answer: What is understanding? Has the word the same meaning for everybody? Does understanding the proof of a theorem consist in examining each of the syllogisms of which it is composed in succession, and being convinced that it is correct and conforms to the rules of the game? Not for the majority [of people]. They want to know not only whether all the syllogisms of a proof are correct, but why they are linked together in one order rather than in another. As long as they appear to be developed by caprice, and not by an intelligence constantly conscious of the end to be attained, they do not think they have understood. At first they still perceive the evidence that is placed before their eyes, but, as they are connected by too attenuated a thread with those that precede and those that follow, they pass without leaving a trace in their brains, and are immediately forgotten.

[In teaching we would like to convey mathematical ideas to students, but the problem is that often we have to adapt to their needs. Is there a fixed mathematics curriculum for all?]
Question: The engineer must receive a complete mathematical training, but of what use is it to him, except to enable him to see the different aspects of things and to see them quickly? [3, essay 'Les définitions mathématiques et l'enseignement']
Answer: He has no time to split hairs. In the complex physical objects that present themselves to him, he must promptly recognize the point where he can apply the mathematical instruments we have put in his hands. The principal aim of mathematical education is to develop certain faculties of the mind, and among these intuition is not the least precious. It is through it that the mathematical world remains in touch with the real world. The practitioner will always need it, and for every pure geometrician there must be a hundred practitioners.

[You have been active in science but also in reaching out to the general public in lectures and essays. Is it not difficult that so many people are doubtful about the value of mathematics?]
Question: People often ask what the use is of mathematics and if these delicate constructions that emerge completely out of our mind are not artificial and created by a whim? [2, essay 'L'analyse et la physique']
Answer: Among the people who ask this question, I make a distinction. Some down-to-earth people are asking only from us a way to make money; those people do not merit an answer. It would be better to ask those who spend their time to become rich, what good it is to neglect at the same time art and science, the only things that enable the souls to enjoy themselves. By the way, science that is only concerned with applications, is impossible; results are only productive when they are connected to each other.

Other people are interested only in nature and they ask us if we are able to improve our knowledge of it. Mathematics has three purposes. It delivers an instrument for the study of nature, but this is not all. It has a philosophical purpose and, I dare say, an aesthetic purpose. These purposes can not be separated and the best way to achieve one purpose is to aim at the other ones, or at least not to loose sight of them.

[Funding of science is an issue that remains opportune at all times.]
Question: Governments and parliaments find astronomy an expensive science; is this correct? [2, essay 'L'astronomie']
Answer: Politicians should have kept a certain rudimentary idealism. Astronomy is useful because it lifts us above ourselves, it is useful because it is great, it is useful because it is beautiful. Astronomy has facilitated the results of other, more directly useful sciences because it has made us capable of understanding nature. The success of astronomy has been that nature follows laws, these laws are unavoidable, one can not ignore them.

[Does funding of science imply a specific political choice of the scientists?]
Question: Should scientists with political interest fight or support the government? [In periodical *La Revue Bleue*, 1904]
Answer: Well, this time I have to excuse myself; everybody will have to choose according to his conscience. I think that not everybody will cast the same vote, and I see no reason to complain about this. If scientists take part in politics, they should take part in all parties, and it is indeed necessary that they be present in the strongest party. Science needs money, and it should not be such that the people with power can say, science, that is the enemy.

Here ends our interview with the amazingly creative professor Poincaré. He was aware of the extent and the importance of his own achievements, but this did not make him rest on his laurels. He never stopped questioning results in the mathematical sciences, their foundations and their role in society until the end of his life.

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