AREA UNDER A CURVE
INTERGATIONAL CALCULUS

Differential Calculus deals with finding the rate of change of one quantity with respect to another…

Integral Calculus deals with precisely the opposite problem:

If we know the rate of change of one quantity with respect to the other, how can we find the relationship between two quantities?

In other words, if you know $\frac{dy}{dx}$ or $f'(x)$…what is $f(x)$?

The tool we used to do this is called the **ANTIDERIVATIVE!**

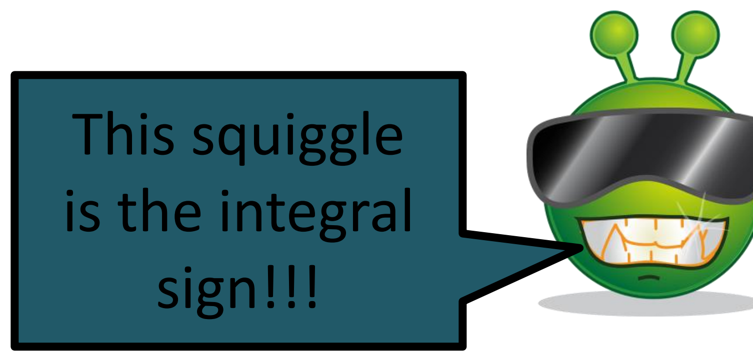
Suppose f is a given function.

An Antiderivative of f is a function F such that

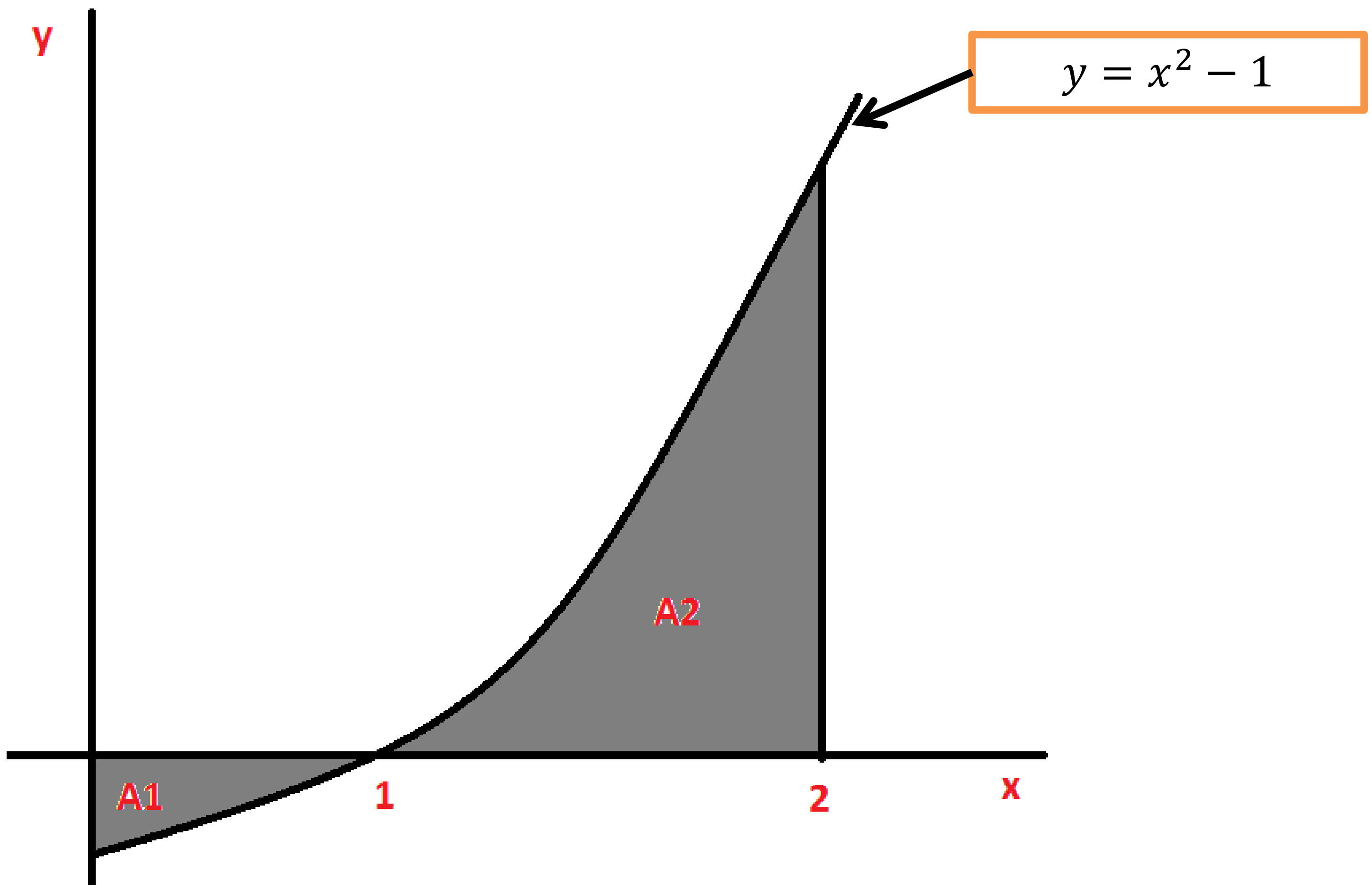
$$F' = f.$$

Thus if f is given, we are asking…what must we differentiate to get f.

$$\int$$



Find the area between the graph $y = x^2 - 1$ between 0 & 2



$$Total\ area = A_2 + A_1$$

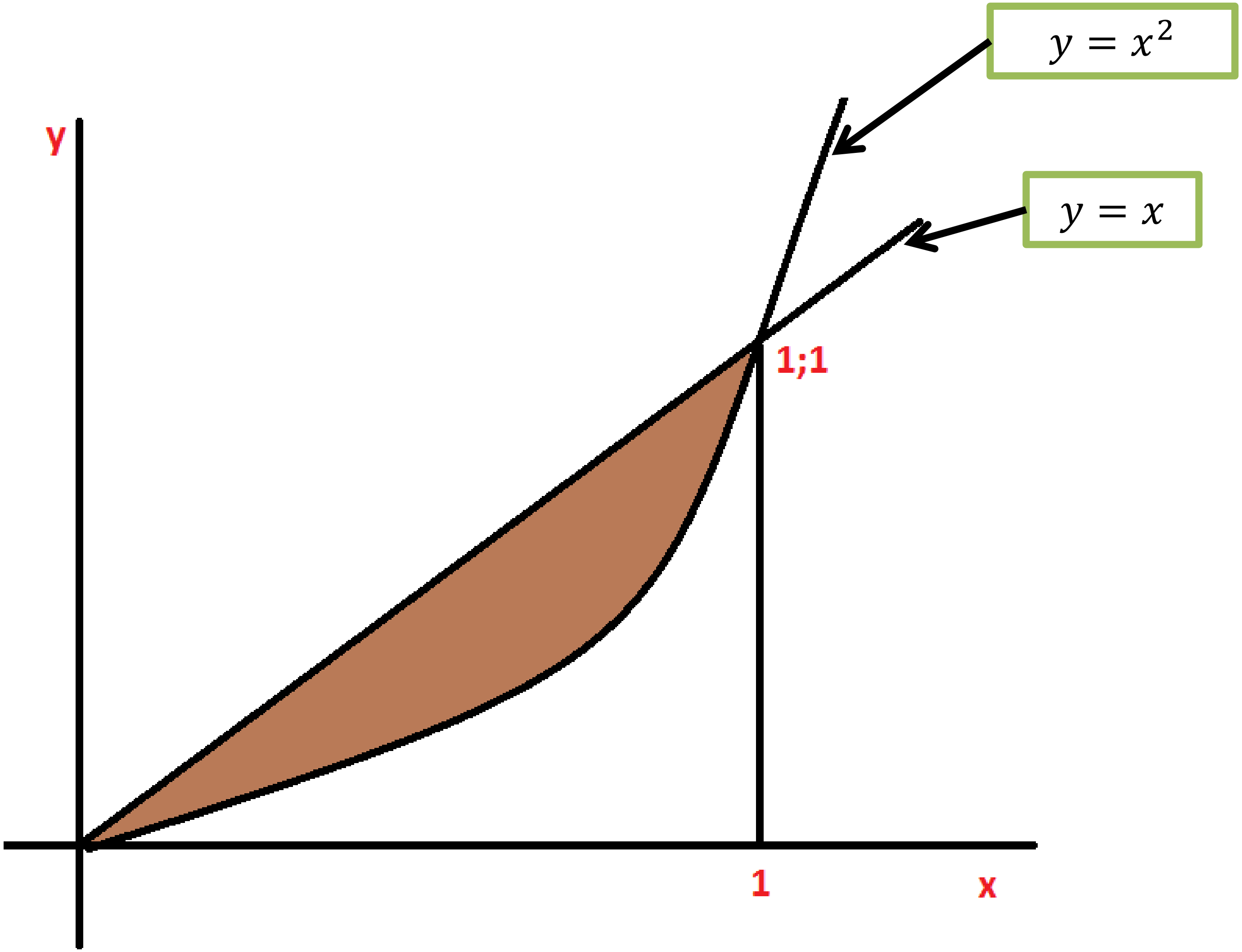
$$= \int_1^2 (x^2 - 1)dx - \int_0^1 (x^2 - 1)dx$$

$$= \left(\frac{1}{3}x^3 - x\right)\Big|_1^2 - (\frac{1}{3}x^3 - x)\Big|_0^1$$

$$= \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right) - (\frac{1}{3} - 1)$$

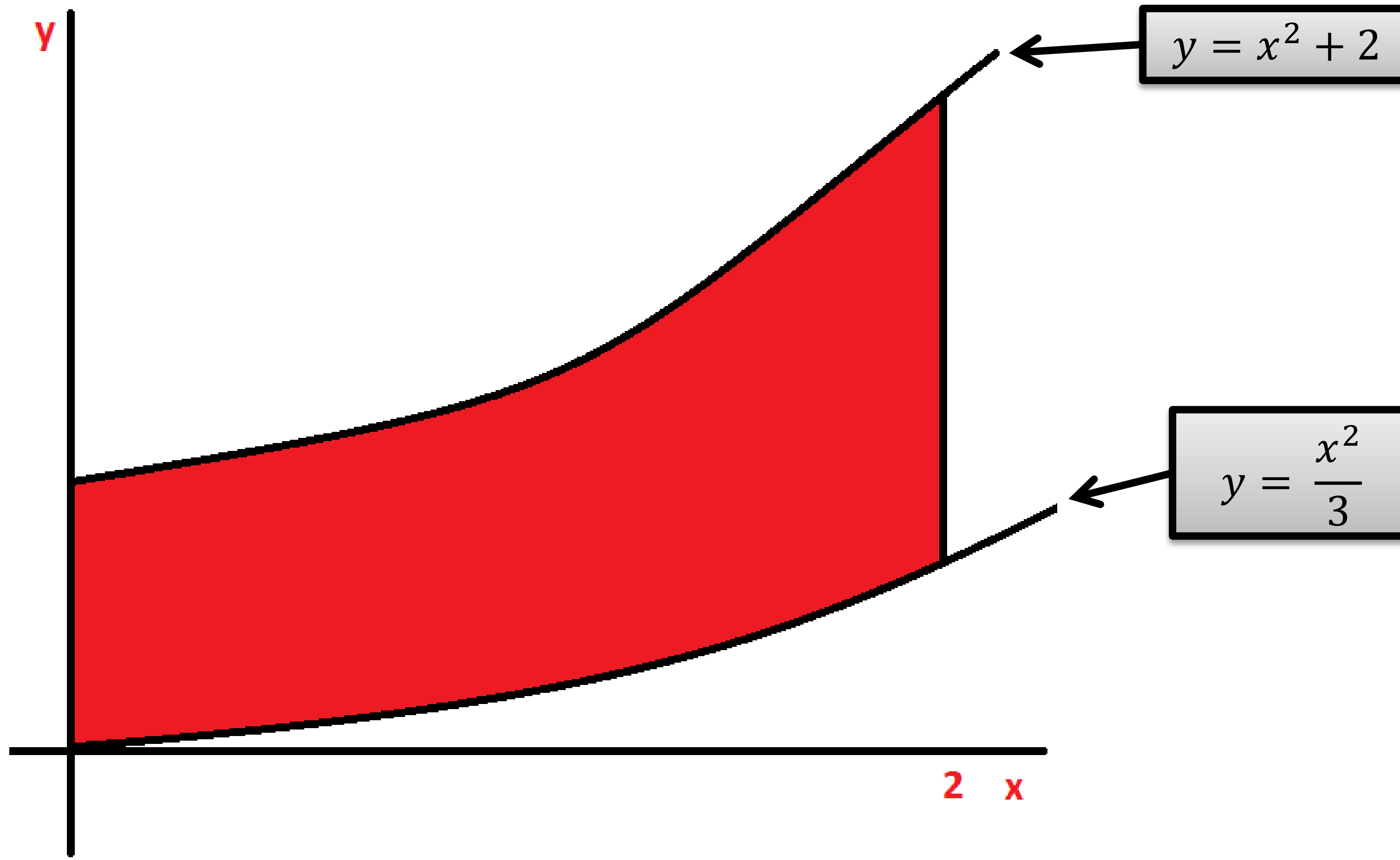
$$= 2$$

Find the area enclosed by the graphs of $y = x$ & $y = x^2$



$$Area = \int_0^1 x\,dx - \int_0^1 x^2 dx$$
$$= \int_0^1 \left(x - x^2\right) dx$$
$$= \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$$
$$= \frac{1}{6}$$

Find the area between the curves $y = x^2 + 2$ & $y = \frac{x^2}{3}$



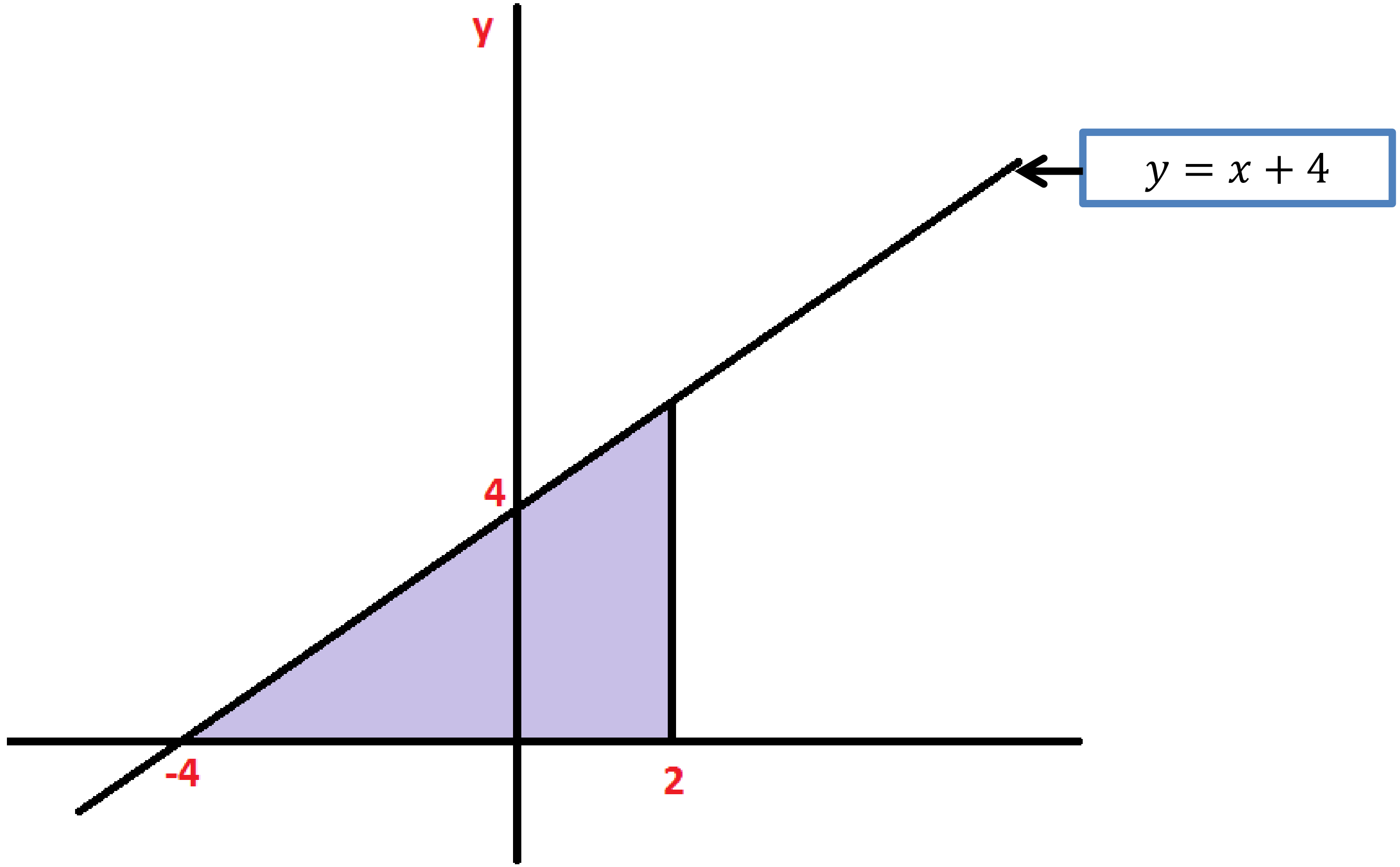
$$Area = \int_0^2 \left(x^2 + 2 - \frac{x^2}{3}\right) dx$$

$$= \int_0^2 \left(\frac{2x^2}{9} + 2x\right)$$

$$= \left(\frac{2x^3}{9} + 2x\right) |_0^2$$

$$= \frac{52}{9}$$

Find the area…



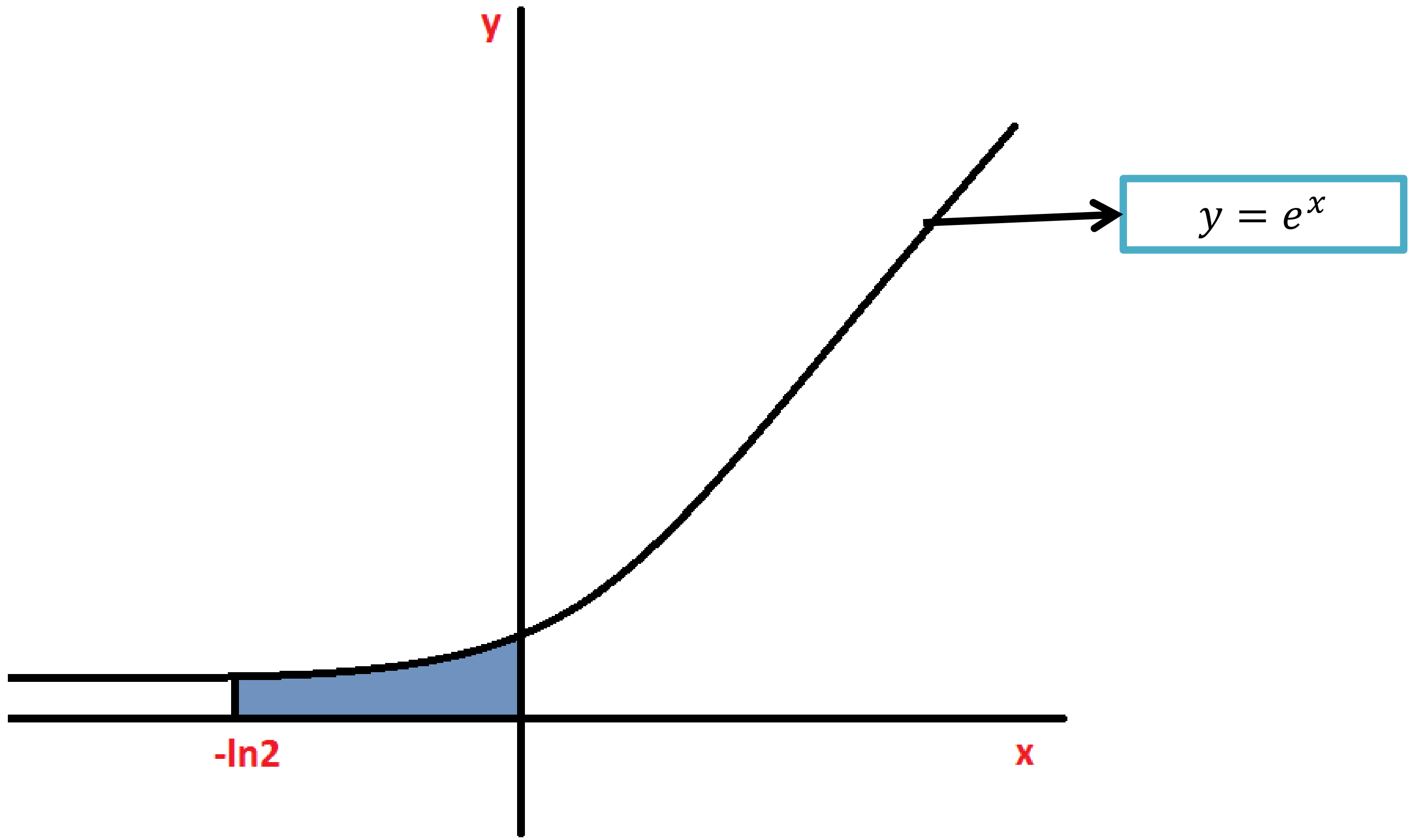
$$Area = \int_{-4}^{2} (x + 4) dx$$

$$= (\frac{1}{2}x^2 + 4x)|_{-4}^{2}$$

$$= (2 + 8) - (8 - 16)$$

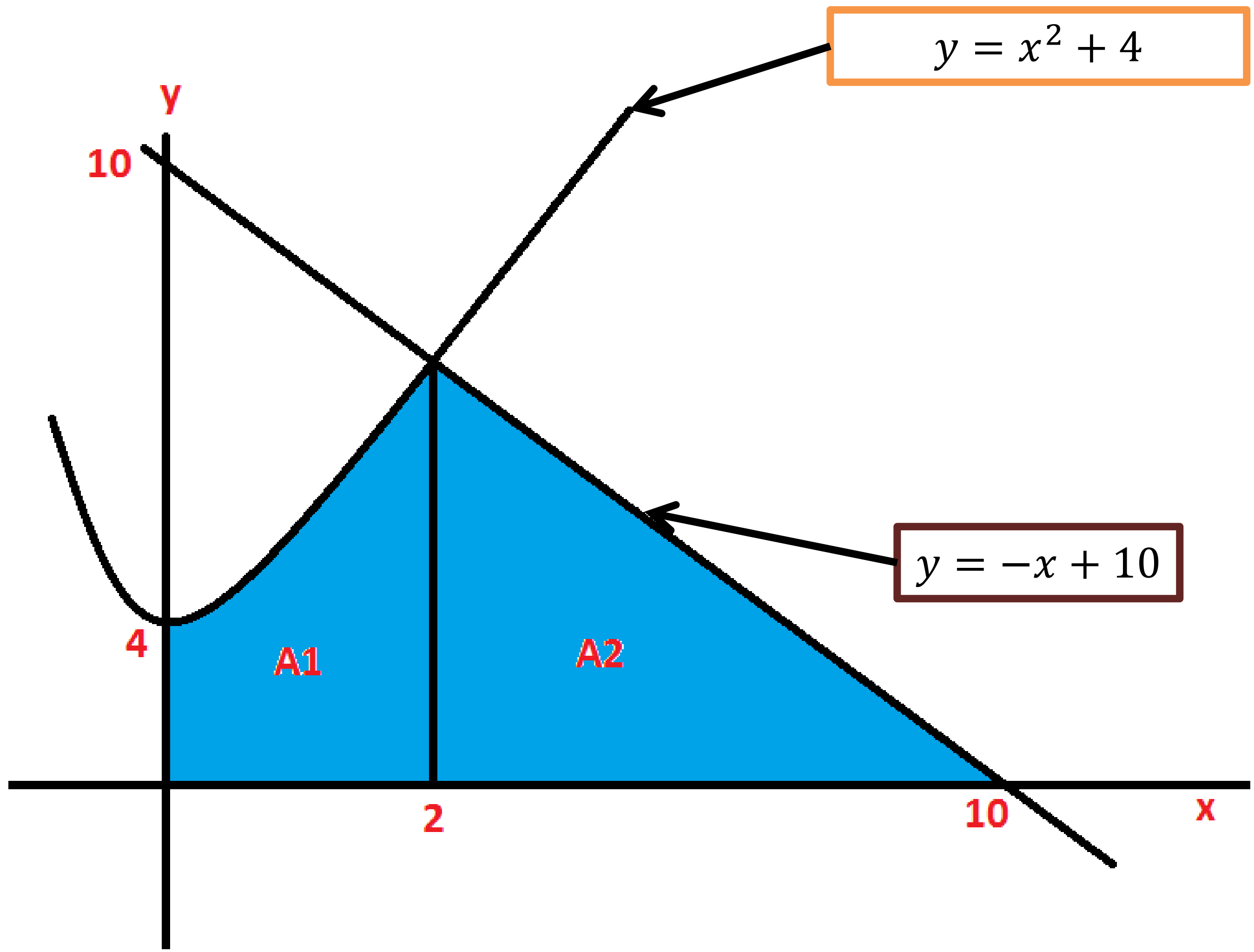
$$= 18$$

Find the area…



$$Area = \int_{\frac{ln1}{2}}^{0} e^x dx$$
$$= e^x|_{\frac{ln1}{2}}^{0}$$
$$= e^0 - e^{\frac{ln1}{2}}$$
$$= 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

Find the area…

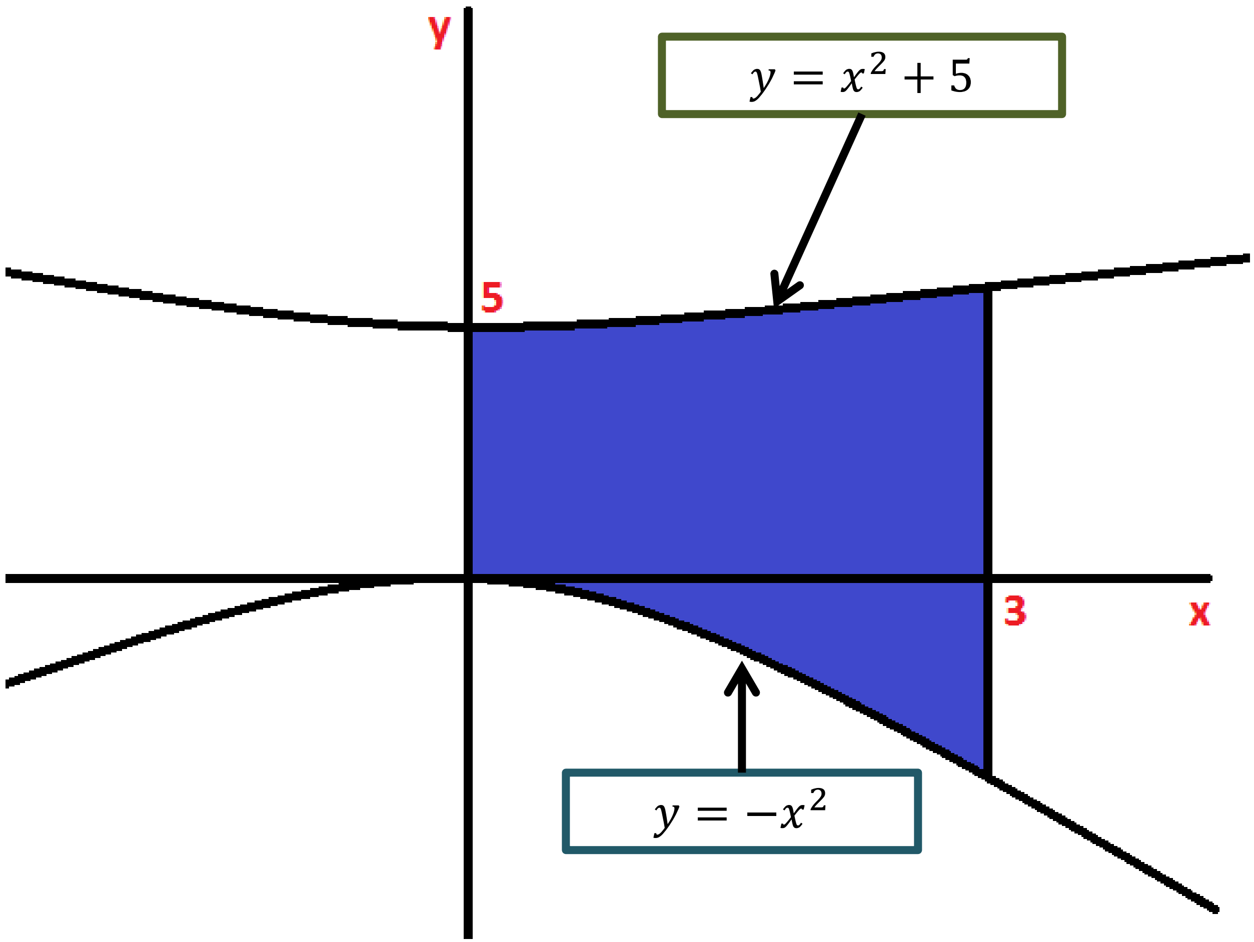


$$A_1 = \int_0^2 (x^2 + 4)dx$$
$$= \left(\frac{1}{3}x^3 + 4x)\right)|_0^2$$
$$= \frac{8}{3} + 8$$
$$= \frac{32}{3}$$

$$A_2 = \int_2^{10} (-x + 10)dx$$
$$= \left(-\frac{1}{2}x^2 + 10x\right)|_0^2$$
$$= -50 + 100 + 2 - 20$$
$$= 32$$

$$Total\ area = A_1 + A_2$$
$$= \frac{32}{3} + 32$$
$$= \frac{128}{3}$$

Find the area...



We simplify…

$$Area = \int_0^3 (x^2 + 5) - (-x^2)dx$$

$$= \int_0^3 (2x^2 + 5)dx$$

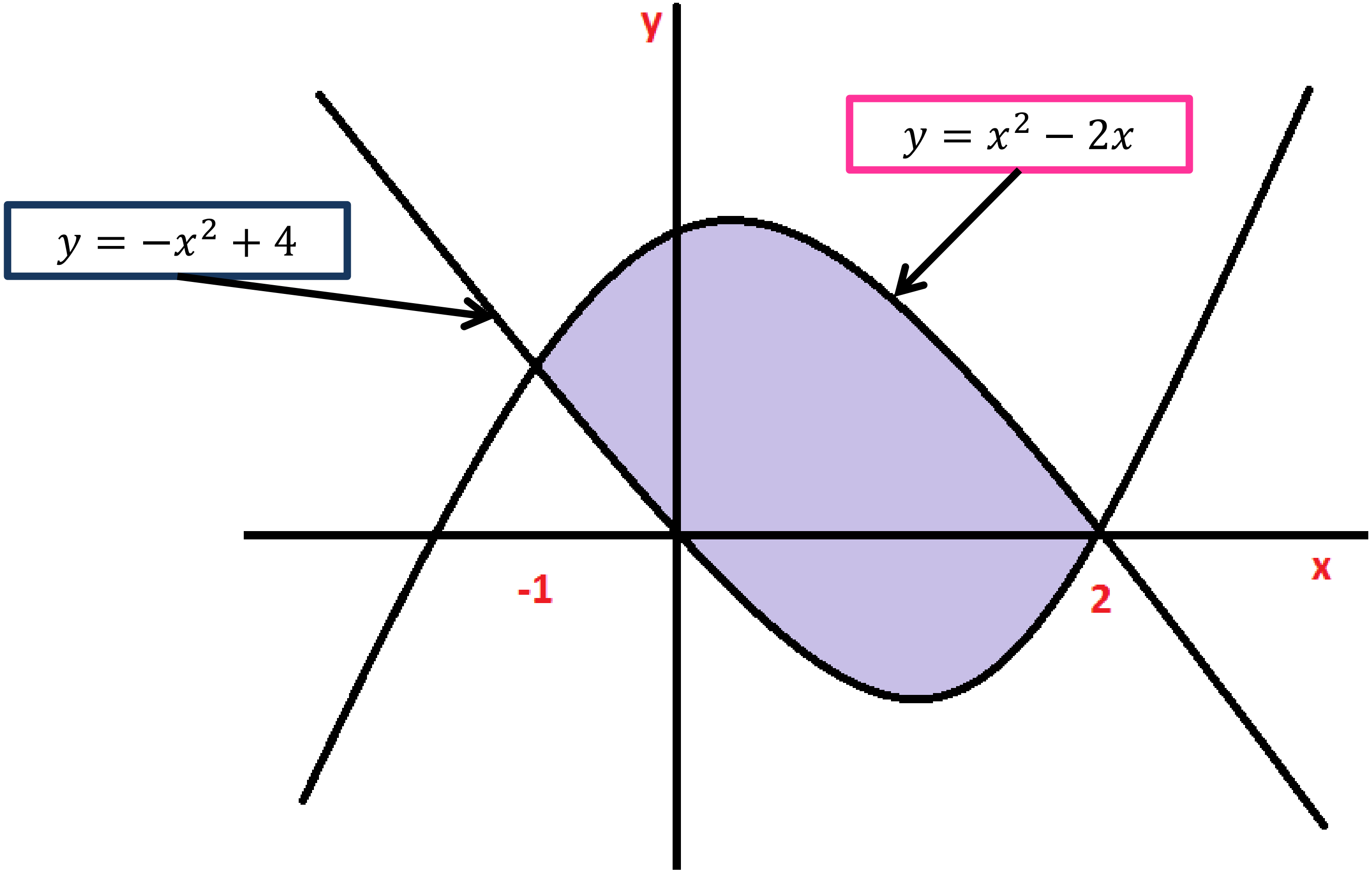
…Now we integrate

$$= \left(\frac{2}{3}x^3 + 5x\right)\Big|_0^3$$

$$= 18 + 15$$

$$= 33$$

Find the area…



$$Area = \int_{-1}^{2} \left[\left(-x^2 + 4\right) - \left(x^2 - 2x\right)\right] dx$$

$$= \int_{-1}^{2} \left(-x^2 + 4 - x^2 + 2x\right) dx$$

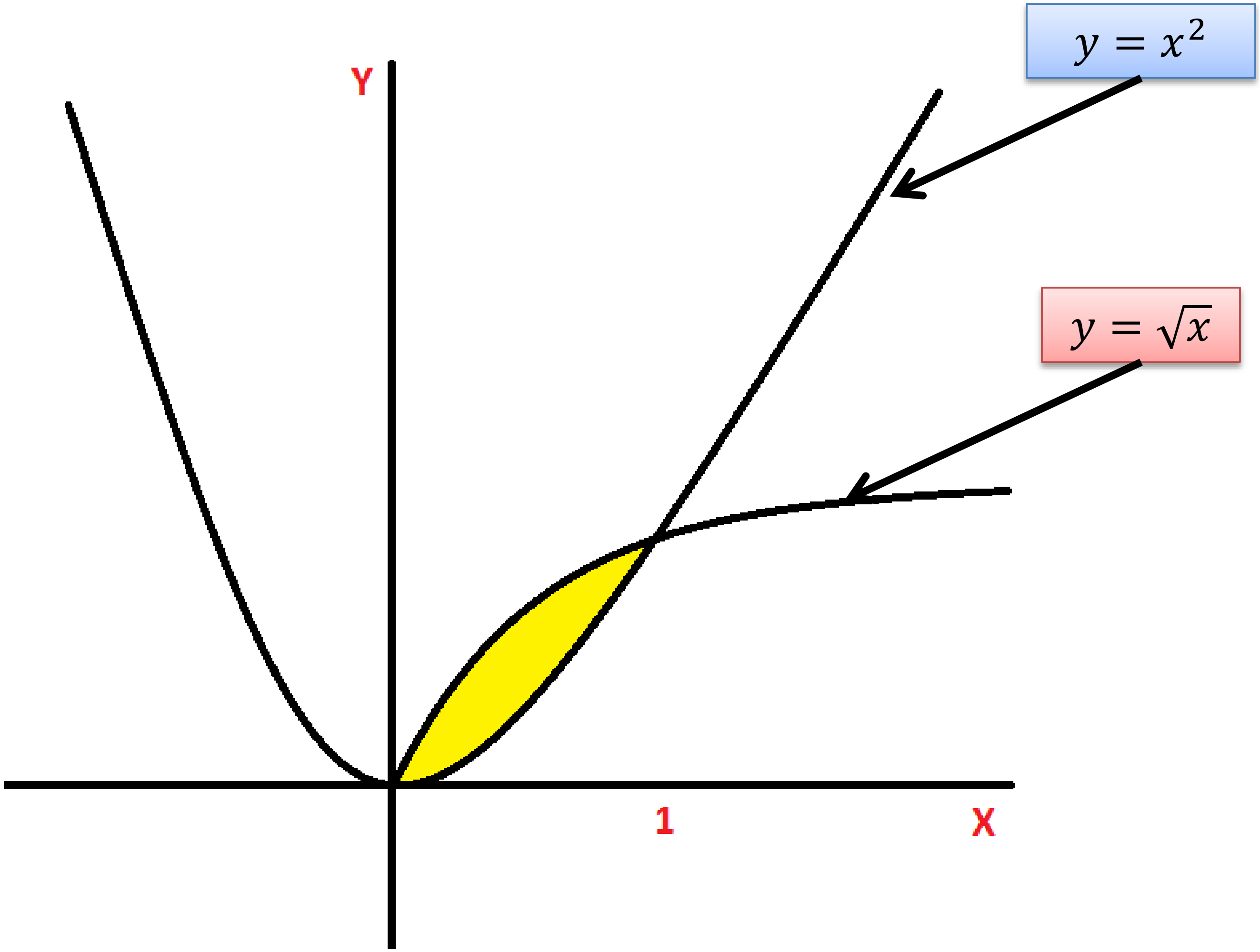
$$= \int_{-1}^{2} \left(-2x^2 + 2x + 4\right) dx$$

$$= \left(-\frac{2}{3}x^3 + x^2 + 4x\right) \Big|_{-1}^{2}$$

$$= -\frac{16}{3} + 4 + 8 - \frac{2}{3} - 1 + 4$$

$$= 9$$

Find the area…



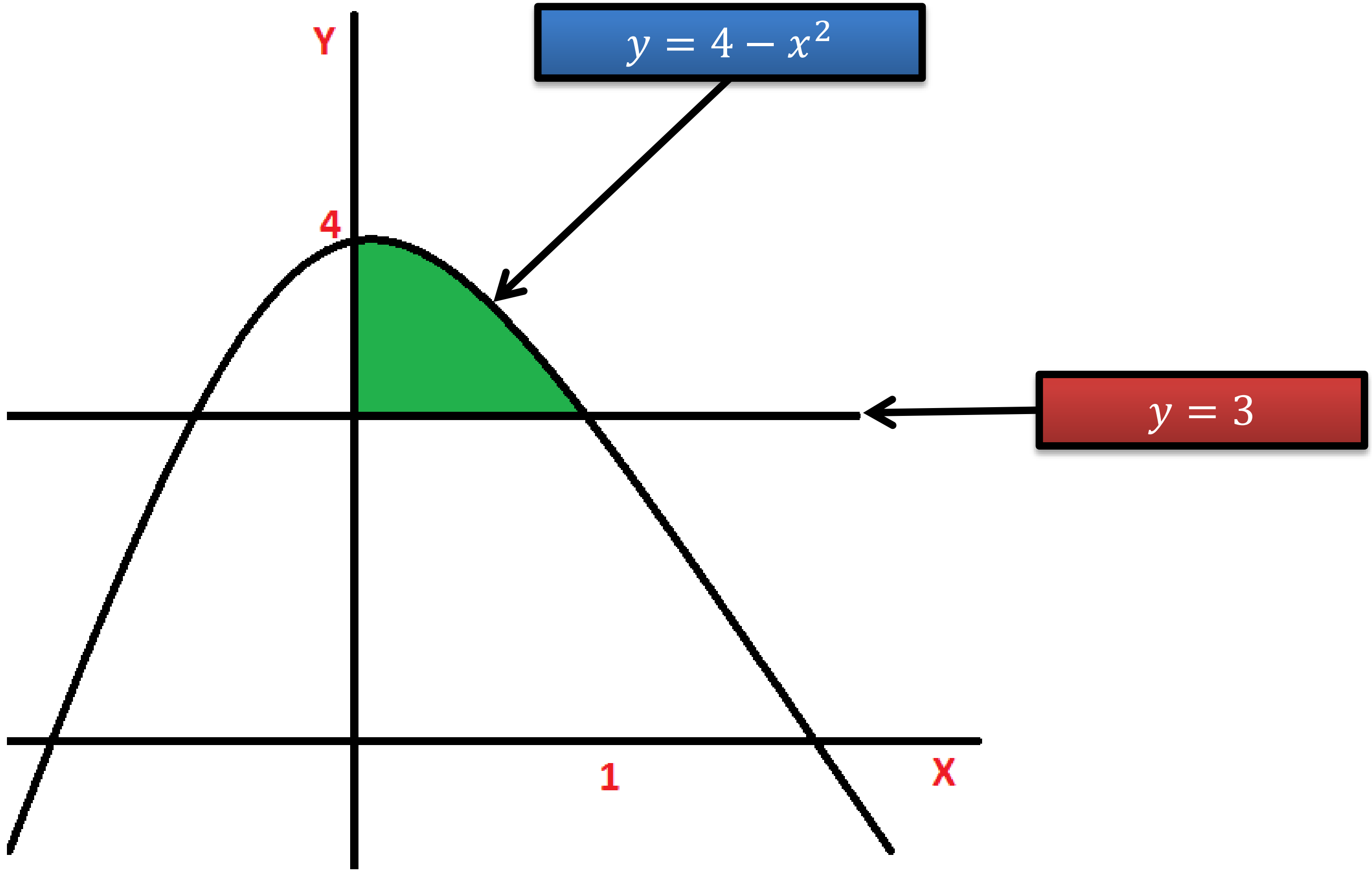
$$Area = \int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$= \left(\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{3} x^3\right) \Big|_0^1$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}$$

Find the area…



$$Area = \int_0^1 (4 - x^2 - 3)dx$$

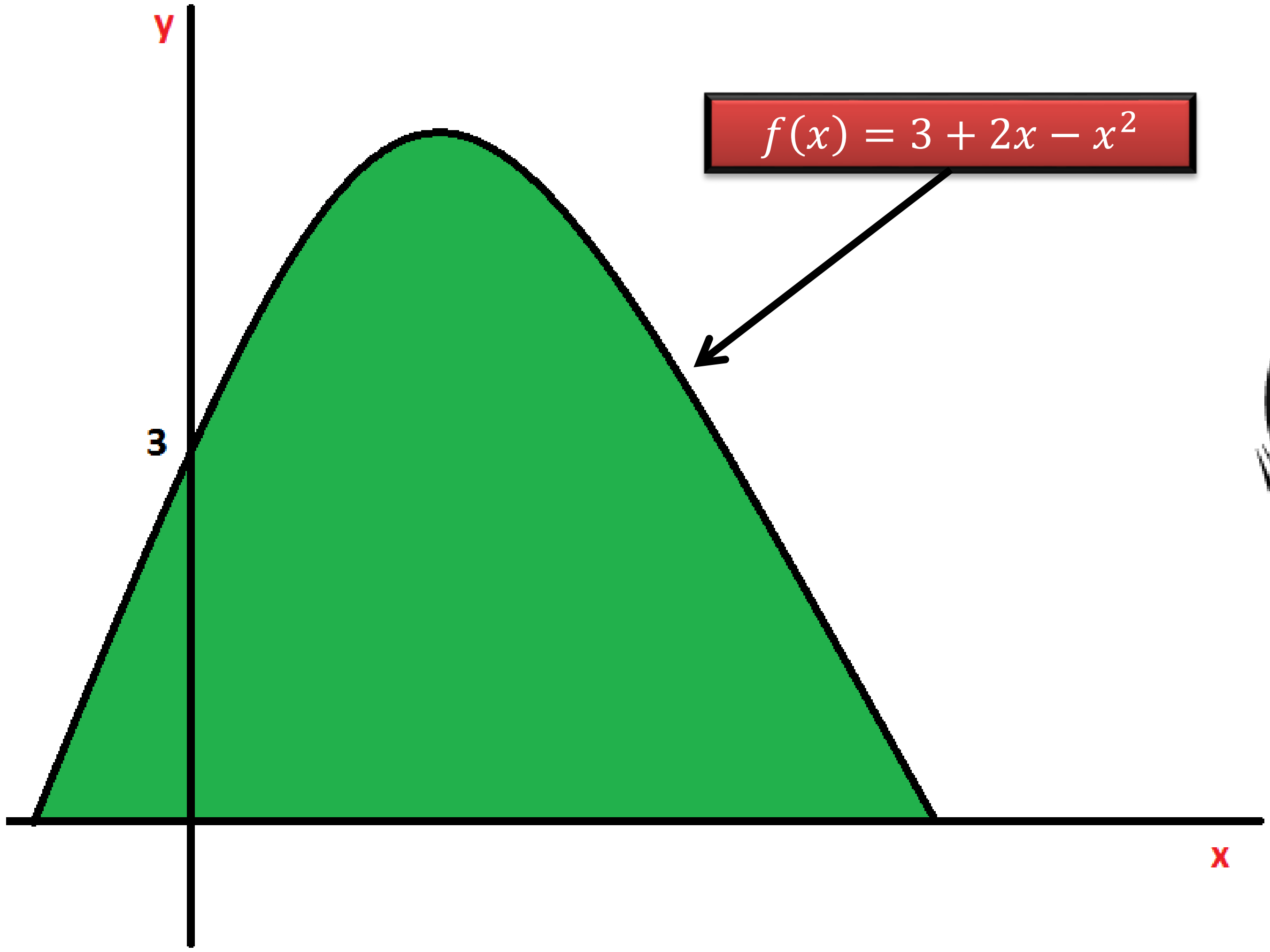
$$= \int_0^1 (1 - x^2)dx$$

$$= \left(x - \frac{1}{3}x^3\right)\Big|_0^1$$

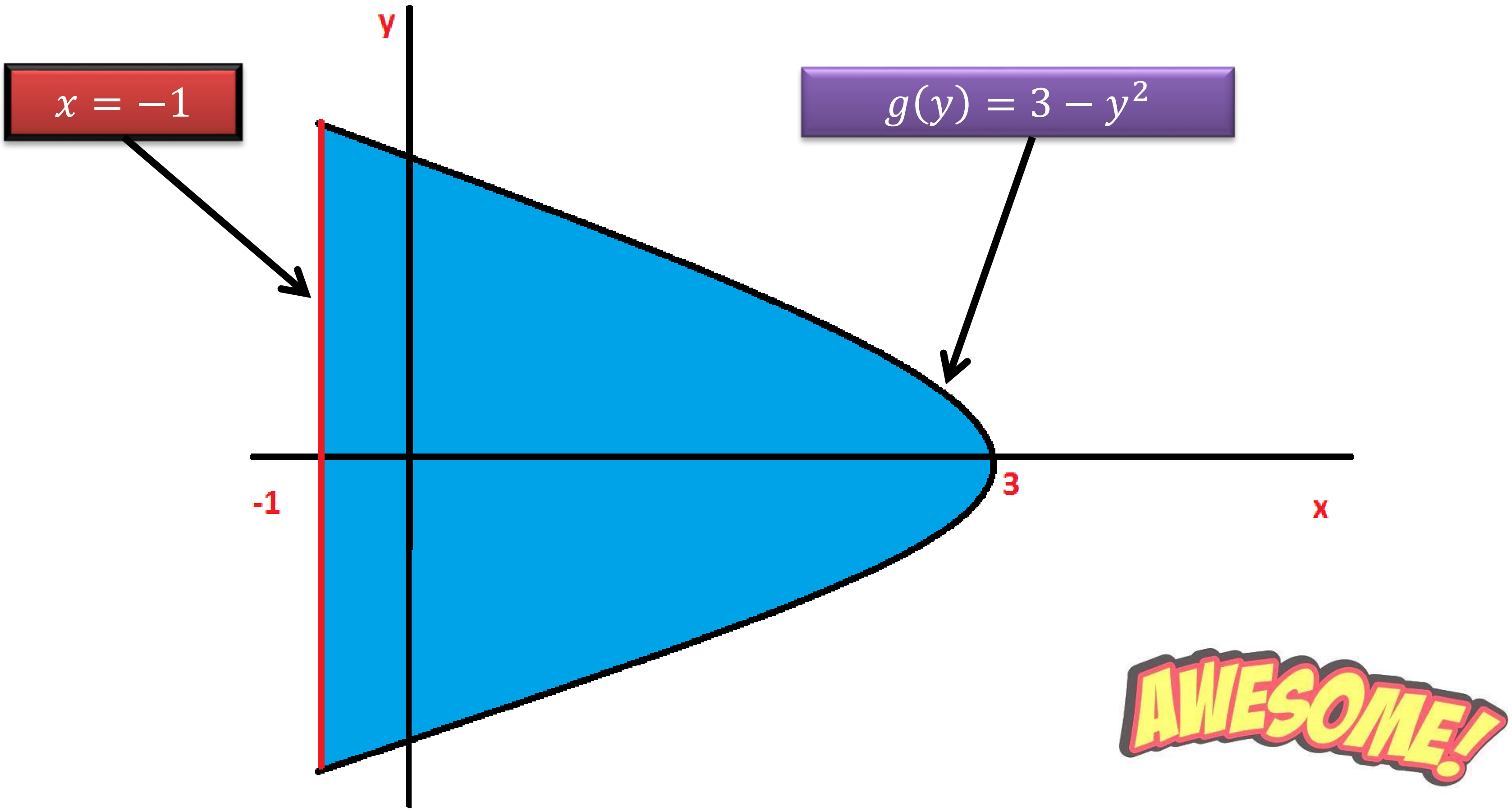
$$Area = \frac{2}{3}$$

Lets do some challenging examples

Determine the area below $f(x) = 3 + 2x - x^2$

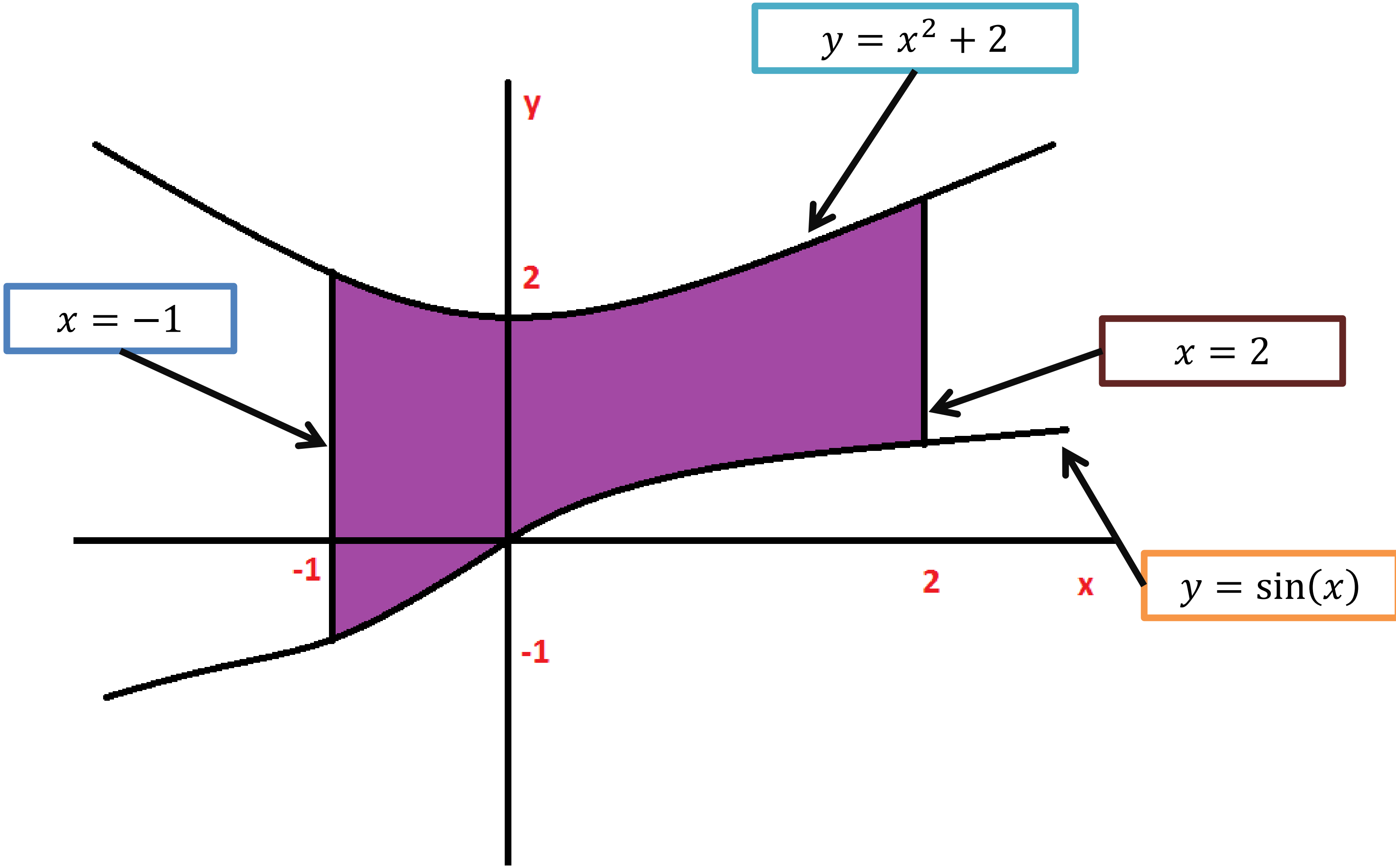


Determine the area to the left of $g(y) = 3 - y^2$ & to the right of $x = -1$



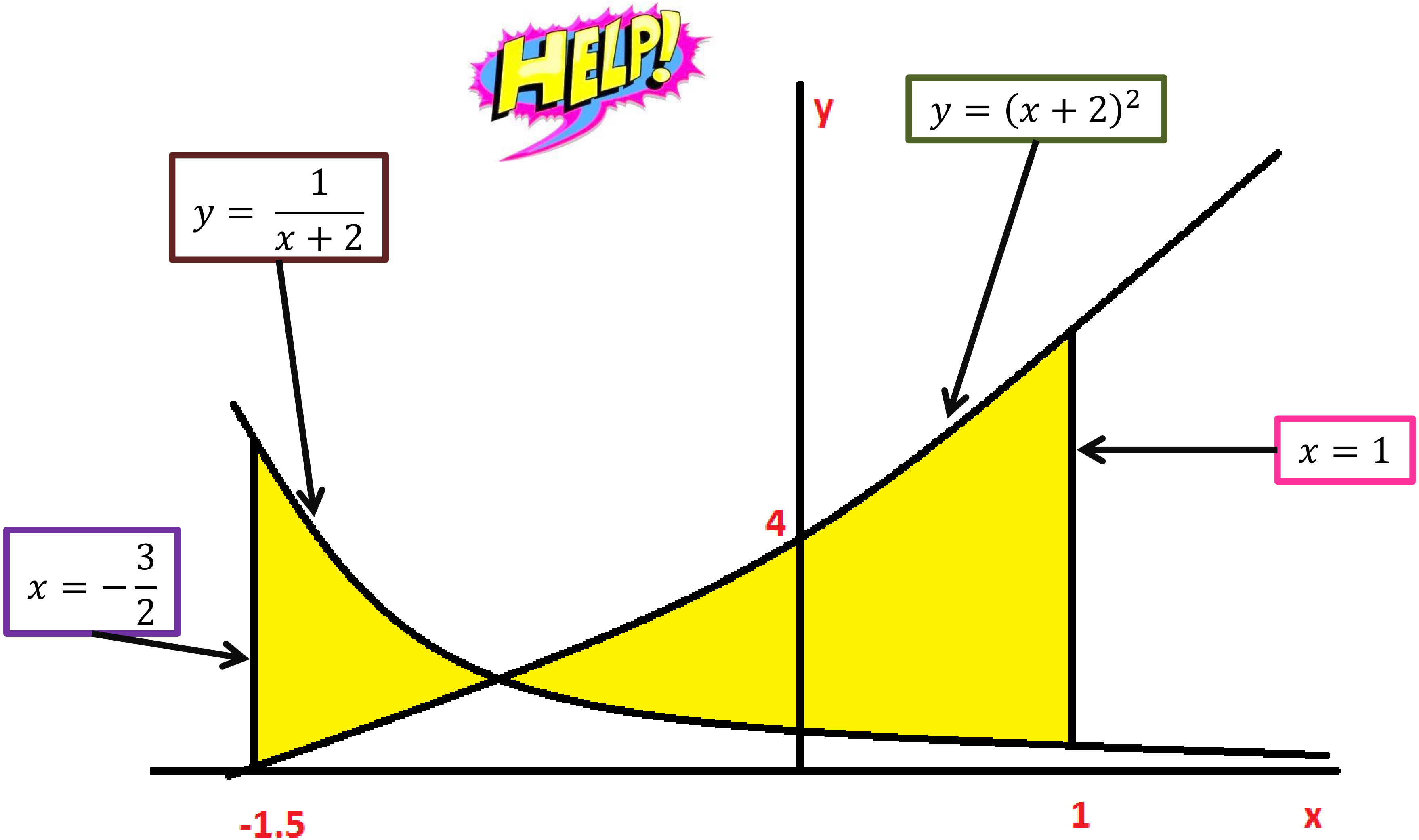
Determine the area of the region bounded by:

$y = x^2 + 2 \ldots y = \sin(x) \ldots x = -1 \ldots x = 2$



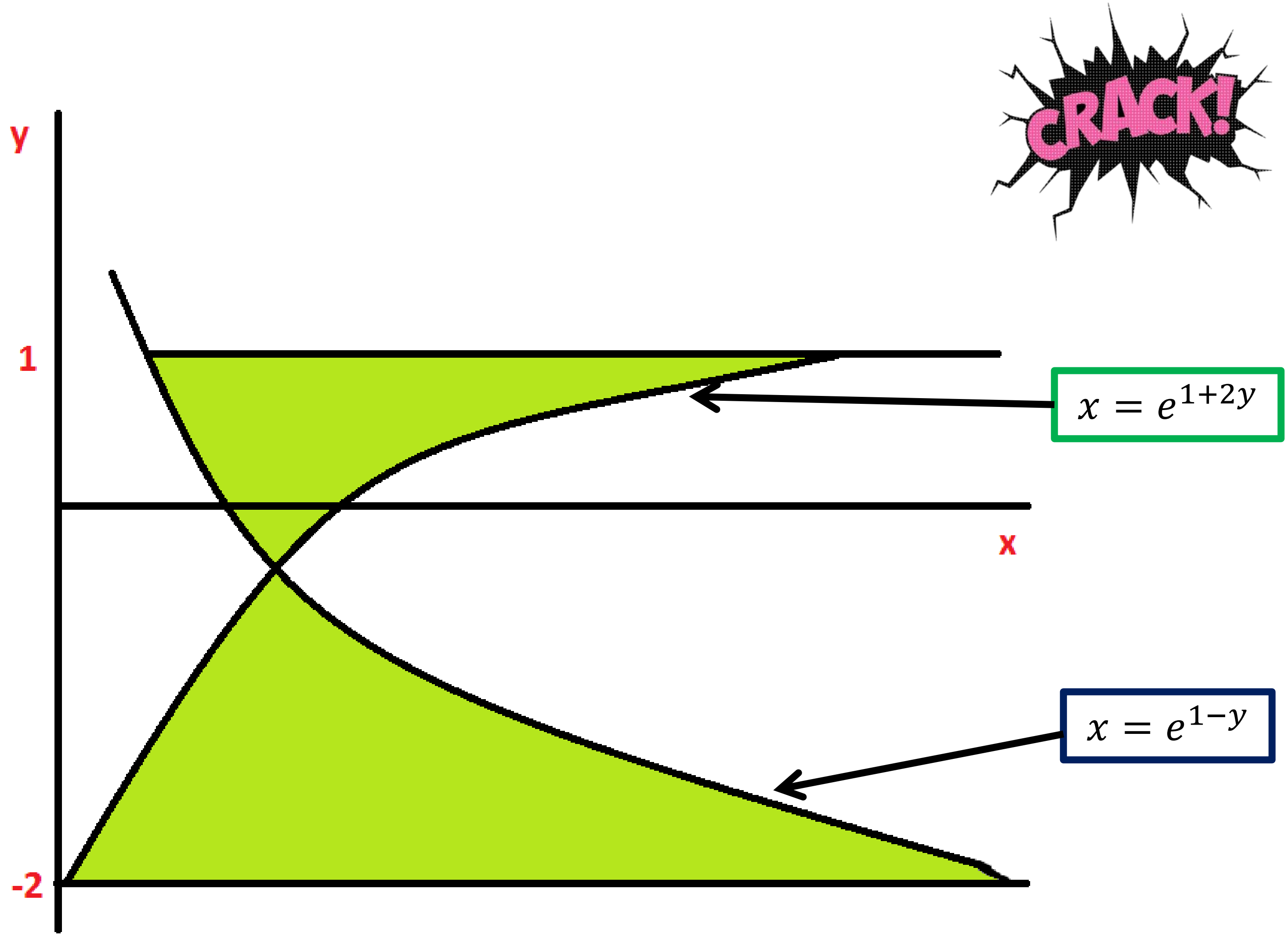
Determine the area of the region bounded by:

$$y = \frac{1}{x+2} \ldots y = (x+2)^2 \ldots x = -\frac{3}{2} \ldots x = 1$$

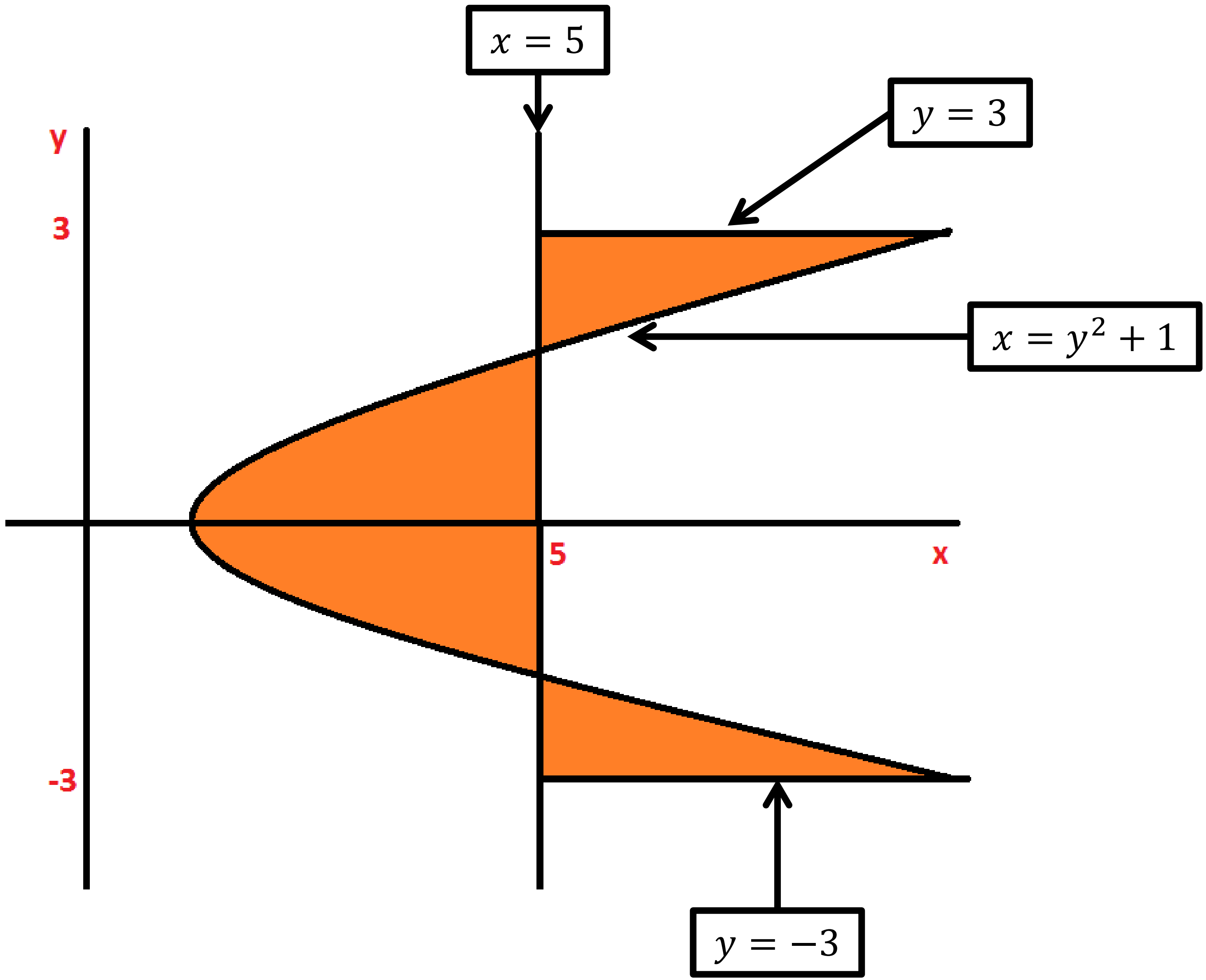


Determine the area of the region bounded by:

$x = e^{1+2y} \ldots x = e^{1-y} \ldots y = -2 \ldots y = 1$



Determine the area of the region bounded by:
$x = y^2 + 1 \ldots x = 5 \ldots y = -3 \ldots y = 3$

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