FINDING THE EQUATION OF A TANGENT TO A CURVE

The derivative or gradient function….

Describes the gradient of a curve

at any point on the curve.

Similarly…it also describes

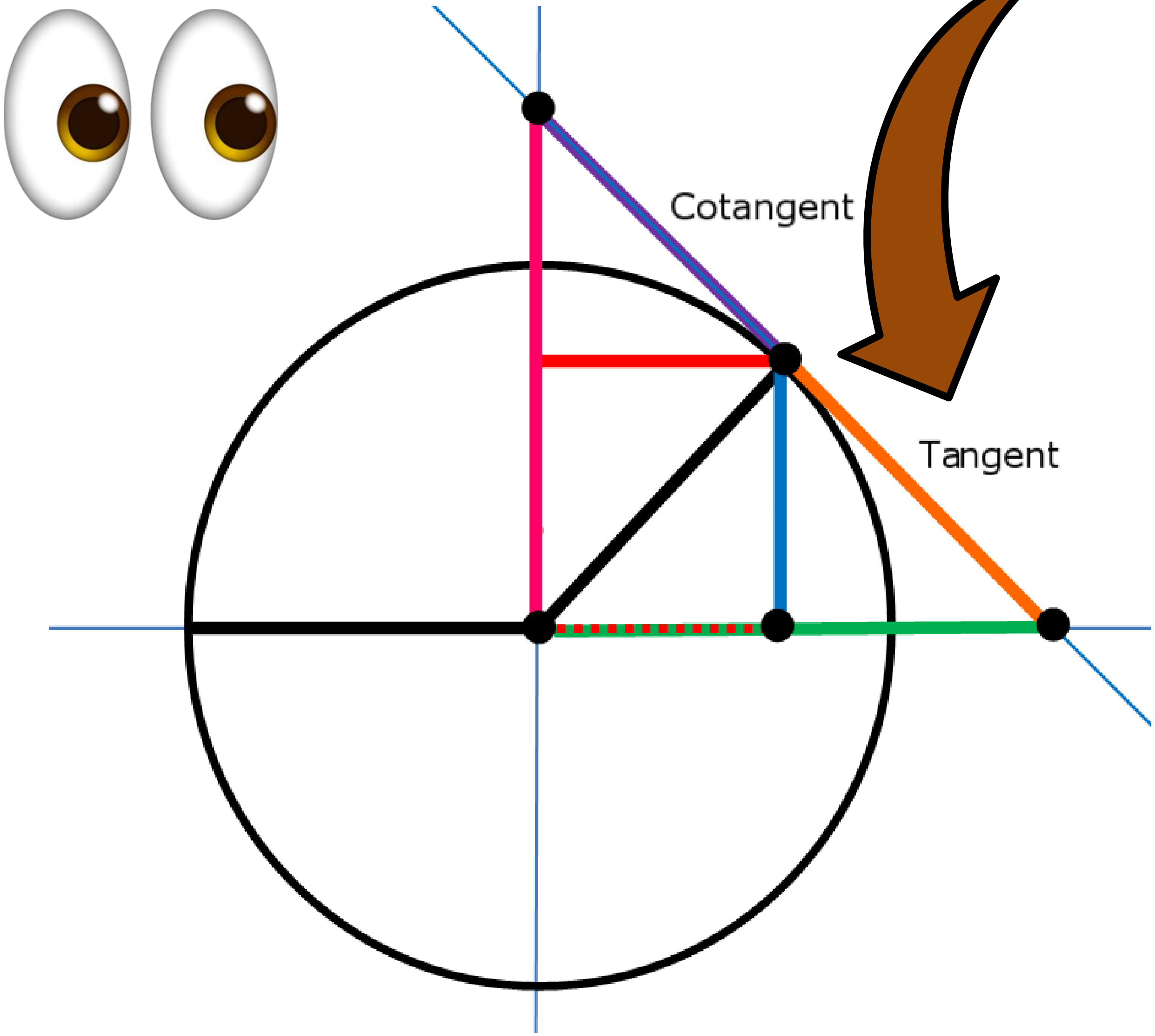
the gradient of a tangent to a curve

At any point on the curve…

…so what's a tangent…?

Well…

Let me refresh your memory…



A TANGENT

is a straight line or plane that touches a curve or curved surface at a point, but if extended... does not cross it at that point...

STEPS…

1. Find the derivative

2. Substitute x co-ordinate of the given point into the derivative to calculate the gradient of the tangent.

3. Substitute the gradient and co-ordinate of the given point into a straight line equation

4. Solve for y

Find the equation of the tangent for the curve $y = 3x^2$ at points (1;3)

Step 1: Find the derivative

$$y = 3x^2$$
$$\boldsymbol{\frac{dy}{dx} = 6x}$$

Step 2: Calculate the gradient at point (1;3)

$$\frac{dy}{dx} = 6x$$
$$= 6(1)$$
$$\boldsymbol{m = 6}$$

Step 3 & 4: Determine equation and solve for y

$$\boldsymbol{y - y_1 = m(x - x_1)}$$
$$y - 3 = 6(x - 1)$$
$$y - 3 = 6x - 6$$
$$y = 6x - 3$$

Solve these...

1. $Given\ g(x) = (x + 2)(2x + 1)^2$
Determine the equation of the tangent at x = -1

2. Determine the equation of the normal to the curve $xy = -4$ at (-1;4)

3. For a particular function g, we know $g'(5) = 2$ & $g(5) = -3$
Write an equation for the line tangent to g at x = 5

4. Consider the function $y = x^3 - x + 5$

a) Find the equation to the line tangent to the curve at point (1;5)

b) Find the equation of the line normal (perpendicular) at the point (1;5)

5. Find the equation of the tangent for $f(x) = \sqrt{x^2 + 3}$ at (-1;2)


ANSWER SECTION

1. $\boldsymbol{Given\ g(x) = (x+2)(2x+1)^2}$

Determine the equation of the tangent at x = -1

Determine the y co-ordinate. Substitute x = =1 in g(x).

$$g(-1) = (-1+2)(2(-1)+1)^2$$
$$= 1$$
$$Tangent\ passes\ through\ (-1; 1)$$

Lets simplify…

$$g(x) = (x+2)(2x+1)^2$$
$$= (x+2)(4x^2+4x+1)$$
$$= 4x^3+4x^2+x+8x^2+8x+2$$
$$= 4x^3+12x^2+9x+2$$

$$g'(x) = 12x^2+24x+9$$
$$sub\ (-1) for\ x\ \ldots$$

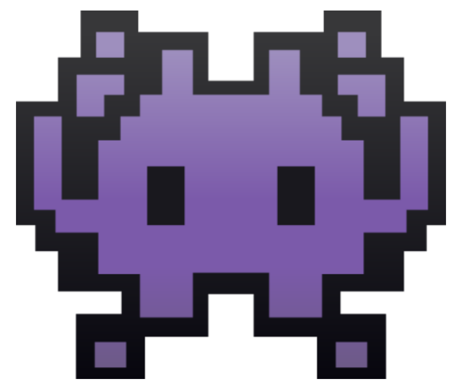
$$g'(-1) = 12(-1)^2+24(-1)+9$$
$$g'(-1) = -3$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -3(x-(-1))$$
$$y = -3x - 3 + 1$$
$$y = -3x - 2$$

2. Determine the equation of the normal to the curve $xy = -4$ at (-1;4)

$$y = -\frac{4}{x}$$
$$y = -4x^{-1}$$
$$\frac{dy}{dx} = 4x^{-2} = \frac{4}{x^2}$$

$$m = \frac{4}{(-1)^2}$$
$$m = 4$$



Use gradient of tangent to calculate gradient of normal

$$m_{tangent} \times m_{normal} = -1$$
$$4 \times m_{normal} = -1$$
$$m_{normal} = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{4}(x - (-1))$$
$$y = -\frac{1}{4}x - \frac{1}{4} + 4$$
$$y = -\frac{1}{4}x + \frac{15}{4}$$

3. For a particular function g, we know $g'(5) = 2$ & $g(5) = -3$

Write an equation for the line tangent to g at x = 5

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 2(x - 5)$$
$$y + 3 = 2x - 10$$
$$y = 2x - 13$$

4. Consider the function $y = x^3 - x + 5$

a) **Find the equation to the line tangent to the curve at point (1;5)**

b) **Find the equation of the line normal (perpendicular) at the point (1;5)**

$$y = x^3 - x + 5$$
$$y' = 3x^2 - 1$$

Substitute 1 in y'

$$y' = 3(1)^2 - 1$$
$$y' = 2$$
$$m = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 2(x - 1)$$
$$y - 5 = 2x - 2$$
$$y = 2x + 3$$

5. Find the equation of the tangent for $f(x) = \sqrt{x^2+3}$ at (-1;2)

$$\boldsymbol{f(x) = \sqrt{x^2+3}}$$

$$\boldsymbol{= \left(x^2+3\right)^{0.5}}$$

$$\boldsymbol{\dots Use\ chain\ rule \dots}$$

$$\boldsymbol{f'(x) = \frac{1}{2}\left(x^2+3\right)^{-0.5}.(2x)}$$

$$\boldsymbol{f'(x) = \frac{x}{\sqrt{x^2+3}}}$$

At (-1;2)…f'(x) = -0.5

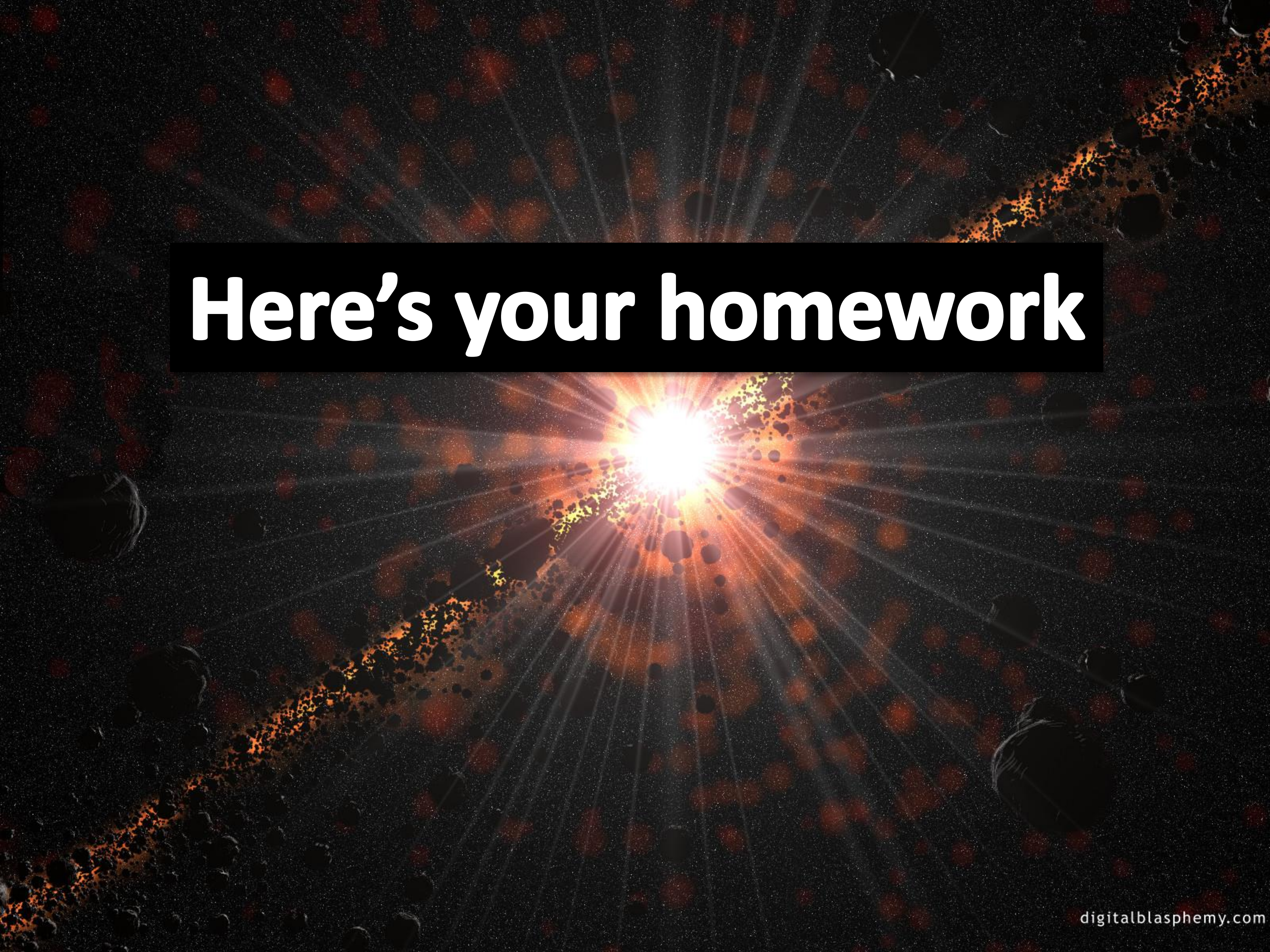
$$y - y_1 = m(x - x_1)$$

$$y - 2 = = -\frac{1}{2}(x - (-1))$$

$$y - 2 = -\frac{1}{2}x - \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

Here’s your homework



For each of the functions below, determine the equations of the tangent.

$a)\ f(x) = x^2 + 3x + 1 \ldots \boldsymbol{at\ x = 0}\ \&\ \boldsymbol{x = 4}$

$b)\ f(x) = 2x^3 - 5x + 4 \ldots \boldsymbol{at\ x = -1}\ \&\ \boldsymbol{x = 1}$

$c)\ f(x) = \tan x \ldots \boldsymbol{at\ x = \frac{\pi}{4}}$

$d)\ f(x) = 3 - x \ldots \boldsymbol{at\ x = -1}\ \&\ \boldsymbol{x = 0}$

HOMEWORK ANSWERS

$a)\ for\ x = 0$

$$y = x^2 + 3x + 1$$
$$y = (0)^2 + 3(0) + 1$$
$$y = 1$$
$$(0; 1)$$

$$f'(x) = 2x + 3$$
$$f'(x) = 2(0) + 3$$
$$f'(x) = 3$$
$$m = 3$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 3(x - 0)$$
$$y - 1 = 3x$$
$$y = 3x + 1$$

$$for\ x = 4 \ldots$$
$$y = x^2 + 3x + 1$$
$$y = (4)^2 + 3(4) + 1$$
$$y = 29$$

$$f'(x) = 2x + 3$$
$$f'(x) = 2(4) + 3$$
$$f'(x) = 11$$
$$m = 11$$

$$y - y_1 = m(x - x_1)$$
$$y - 29 = 11(x - 4)$$
$$y - 29 = 11x - 44$$
$$y = 11x - 15$$

b) $for\ x = 1$

$$f(x) = 2x^3 - 5x + 4$$
$$f(-1) = 2(-1)^3 - 5(-1) + 4$$
$$f(-1) = 7$$
$$(-1; 7)$$

$$f'(x) = 6x^2 - 5$$
$$f'(-1) = 6(-1)^2 - 5$$
$$f'(-1) = 1$$
$$m = 1$$

$$y\ - y_1 = m(x - x_1)$$
$$y - 7 = 1\big(x - (-1)\big)$$
$$y - 7 = x + 1$$
$$y = x + 8$$

$$for\ x = 1$$
$$f(x) = 2x^3 - 5x + 4$$
$$f(1) = 2(1)^3 - 5(1) + 4$$
$$f(1) = 1$$
$$(1; 1)$$

$$f'(x) = 6x^2 - 5$$
$$f'(1) = 6(1)^2 - 5$$
$$f'(1) = 1$$
$$m = 1$$

$$y\ - y_1 = m(x - x_1)$$
$$y - 1 = 1\big(x - (-1)\big)$$
$$y - 1 = x - 1$$
$$y = x$$

c)

$$f(x) = tanx$$
$$f\left(\frac{\pi}{4}\right) = \tan(\frac{\pi}{4})$$
$$= 1$$
$$(\frac{\pi}{4}; 1)$$

$$f'(x) = \sec^2 x$$
$$= \sec^2 \frac{\pi}{4}$$
$$= \left(\frac{1}{cos\frac{\pi}{4}}\right)^2$$
$$f'(x) = 2$$
$$m = 2$$

$$\boldsymbol{y - y_1 = m(x - x_1)}$$
$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$
$$y - 1 = 2x - \frac{2\pi}{4}$$
$$y = 2x - \frac{\pi}{2} + 1$$

d) $for\ x = -2$

$$f(x) = 3 - x$$
$$f(-2) = 3 - (-2)$$
$$f(-2) = 5$$

$$f'(x) = -1$$
$$m = -1$$

$$\boldsymbol{y - y_1 = m(x - x_1)}$$
$$y - 5 = -1(x + 2)$$
$$y - 5 = -x - 2$$
$$y = -x + 3$$

$for\ x = 0$

$$f(x) = 3 - x$$
$$f(0) = 3 - (0)$$
$$f(0) = 3$$

$$f'(x) = -1$$
$$m = -1$$

$$\boldsymbol{y - y_1 = m(x - x_1)}$$
$$y - 3 = -1(x - 0)$$
$$y - 3 = -x$$
$$y = -x + 3$$


…Well that’s about it for now…