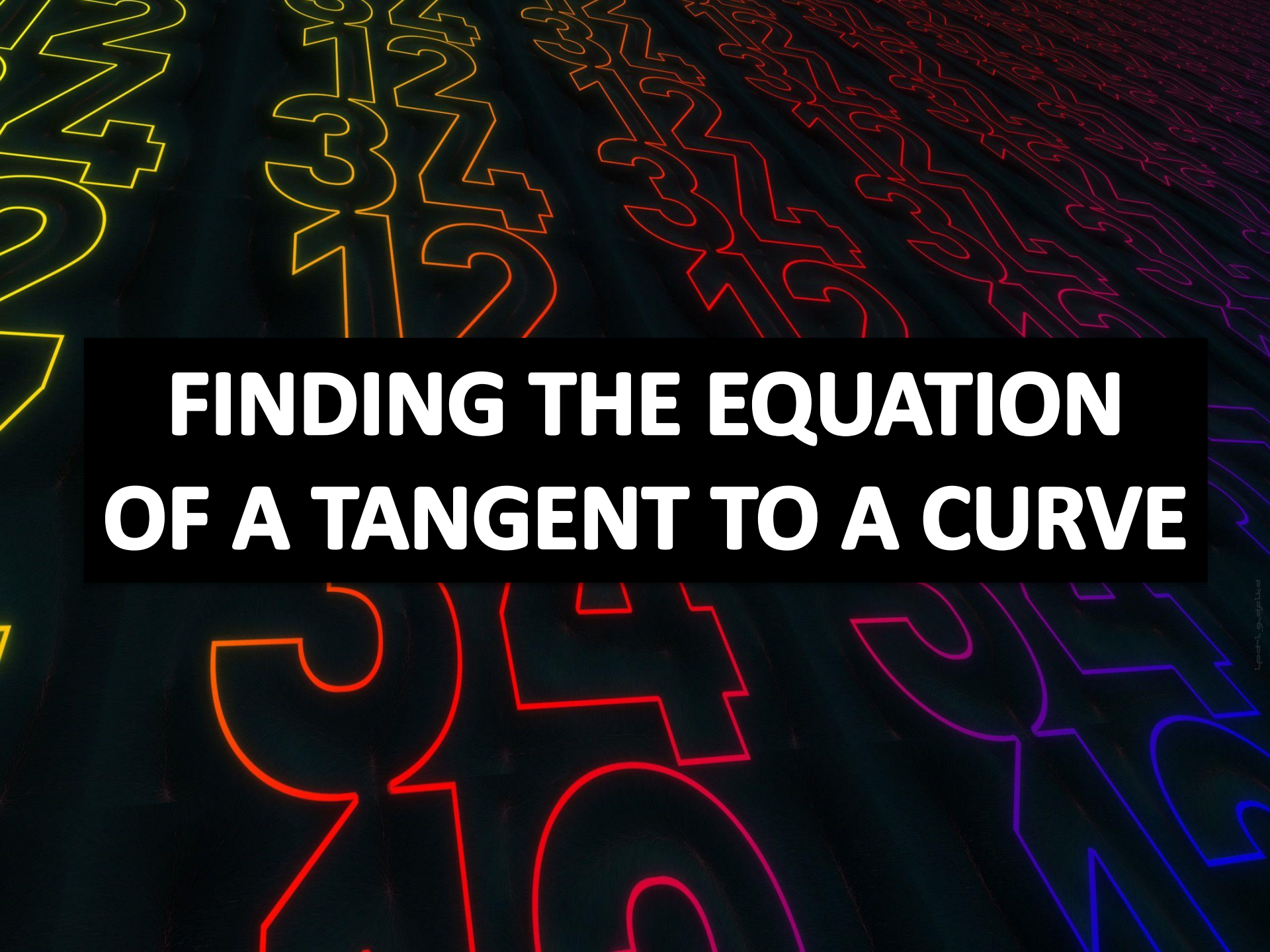


The background is a dark, textured surface with a grid of glowing, stylized numbers and symbols in various colors including yellow, orange, red, and purple. The symbols are reminiscent of mathematical or financial notations, such as dollar signs and numbers, rendered in a jagged, neon-like style.

FINDING THE EQUATION OF A TANGENT TO A CURVE

The derivative or gradient function....

Describes the gradient of a curve

at any point on the curve.



Similarly...it also describes

the gradient of a tangent to a curve

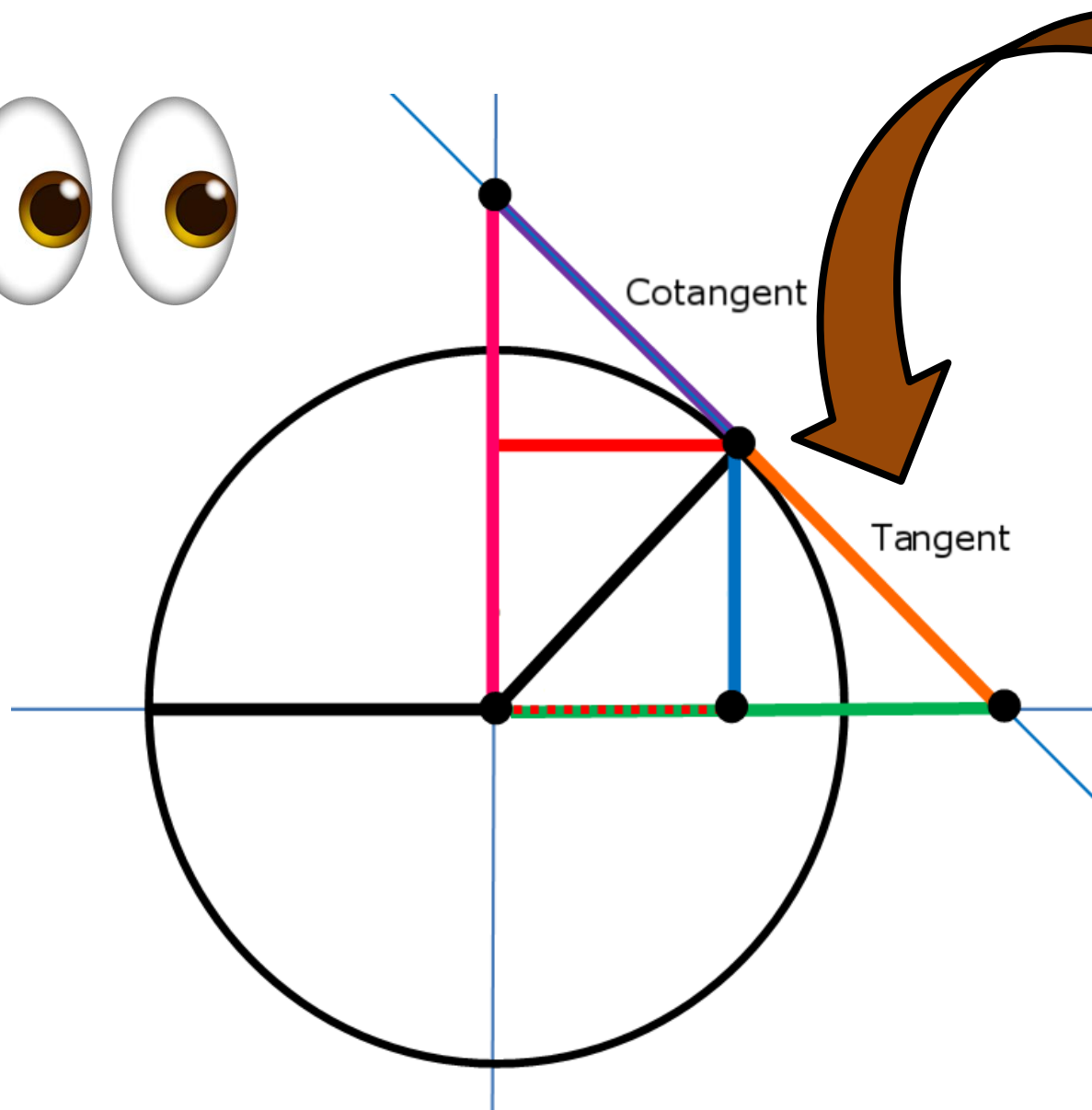
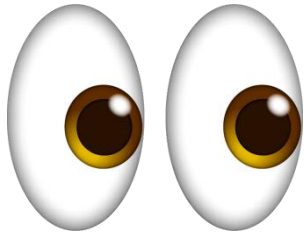
At any point on the curve...

...so what's a tangent...?

Well...

Let me refresh
your memory...





A TANGENT
is a straight
line or plane
that touches
a curve or
curved
surface at a
point, but if
extended...
does not
cross it at
that point...

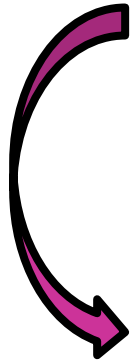
STEPS...

1. Find the derivative

2. Substitute x co-ordinate of the given point into the derivative to calculate the gradient of the tangent.

3. Substitute the gradient and co-ordinate of the given point into a straight line equation

4. Solve for y



Find the equation of the tangent for the curve $y = 3x^2$ at points (1;3)

Step 1: Find the derivative

$$y = 3x^2$$
$$\frac{dy}{dx} = 6x$$

Step 2: Calculate the gradient at point (1;3)

$$\frac{dy}{dx} = 6x$$
$$= 6(1)$$
$$m = 6$$



Step 3 & 4: Determine equation and solve for y

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 6(x - 1)$$
$$y - 3 = 6x - 6$$
$$y = 6x - 3$$

Solve these...



1. Given $g(x) = (x + 2)(2x + 1)^2$

Determine the equation of the tangent at $x = -1$

2. Determine the equation of the normal to the curve $xy = -4$ at $(-1;4)$

3. For a particular function g , we know $g'(5) = 2$ & $g(5) = -3$

Write an equation for the line tangent to g at $x = 5$

4. Consider the function $y = x^3 - x + 5$

- a) Find the equation to the line tangent to the curve at point $(1;5)$**
- b) Find the equation of the line normal (perpendicular) at the point $(1;5)$**

5. Find the equation of the tangent for $f(x) = \sqrt{x^2 + 3}$ at $(-1;2)$



ANSWER SECTION

1. **Given** $g(x) = (x + 2)(2x + 1)^2$

Determine the equation of the tangent at $x = -1$

Determine the y co-ordinate. Substitute $x = -1$ in $g(x)$.

$$\begin{aligned}g(-1) &= (-1 + 2)(2(-1) + 1)^2 \\ &= 1\end{aligned}$$

Tangent passes through $(-1; 1)$

Lets simplify...

$$\begin{aligned}g(x) &= (x + 2)(2x + 1)^2 \\ &= (x + 2)(4x^2 + 4x + 1) \\ &= 4x^3 + 4x^2 + x + 8x^2 + 8x + 2 \\ &= 4x^3 + 12x^2 + 9x + 2\end{aligned}$$

$$\begin{aligned}g'(x) &= 12x^2 + 24x + 9 \\ &\text{sub } (-1) \text{ for } x \dots\end{aligned}$$

$$\begin{aligned}g'(-1) &= 12(-1)^2 + 24(-1) + 9 \\ g'(-1) &= -3\end{aligned}$$

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - 1 &= -3(x - (-1)) \\ y &= -3x - 3 + 1 \\ y &= -3x - 2\end{aligned}$$



2. Determine the equation of the normal to the curve $xy = -4$ at $(-1;4)$

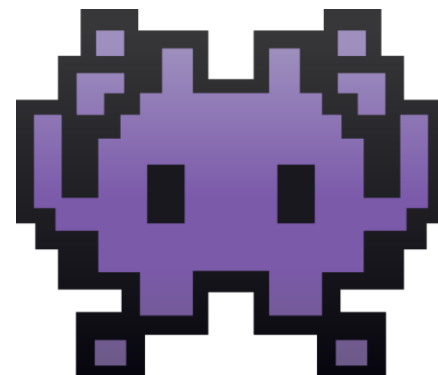
$$y = -\frac{4}{x}$$
$$y = -4x^{-1}$$
$$\frac{dy}{dx} = 4x^{-2} = \frac{4}{x^2}$$

$$m = \frac{4}{(-1)^2}$$
$$m = 4$$

Use gradient of tangent to calculate gradient of normal

$$m_{\text{tangent}} \times m_{\text{normal}} = -1$$
$$4 \times m_{\text{normal}} = -1$$
$$m_{\text{normal}} = -\frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{4}(x - (-1))$$
$$y = -\frac{1}{4}x - \frac{1}{4} + 4$$
$$y = -\frac{1}{4}x + \frac{15}{4}$$



3. For a particular function g , we know $g'(5) = 2$ & $g(5) = -3$

Write an equation for the line tangent to g at $x = 5$

$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - (-3) &= 2(x - 5) \\y + 3 &= 2x - 10 \\y &= 2x - 13\end{aligned}$$



4. Consider the function $y = x^3 - x + 5$

- a) Find the equation to the line tangent to the curve at point (1;5)**
- b) Find the equation of the line normal (perpendicular) at the point (1;5)**

$$y = x^3 - x + 5$$
$$y' = 3x^2 - 1$$

Substitute 1 in y'

$$y' = 3(1)^2 - 1$$
$$y' = 2$$
$$m = 2$$



$$y - y_1 = m(x - x_1)$$
$$y - 5 = 2(x - 1)$$
$$y - 5 = 2x - 2$$
$$y = 2x + 3$$

5. Find the equation of the tangent for $f(x) = \sqrt{x^2 + 3}$ at $(-1;2)$

$$f(x) = \sqrt{x^2 + 3}$$

$$= (x^2 + 3)^{0.5}$$

... *Use chain rule* ...

$$f'(x) = \frac{1}{2}(x^2 + 3)^{-0.5} \cdot (2x)$$

$$f'(x) = \frac{x}{\sqrt{x^2 + 3}}$$

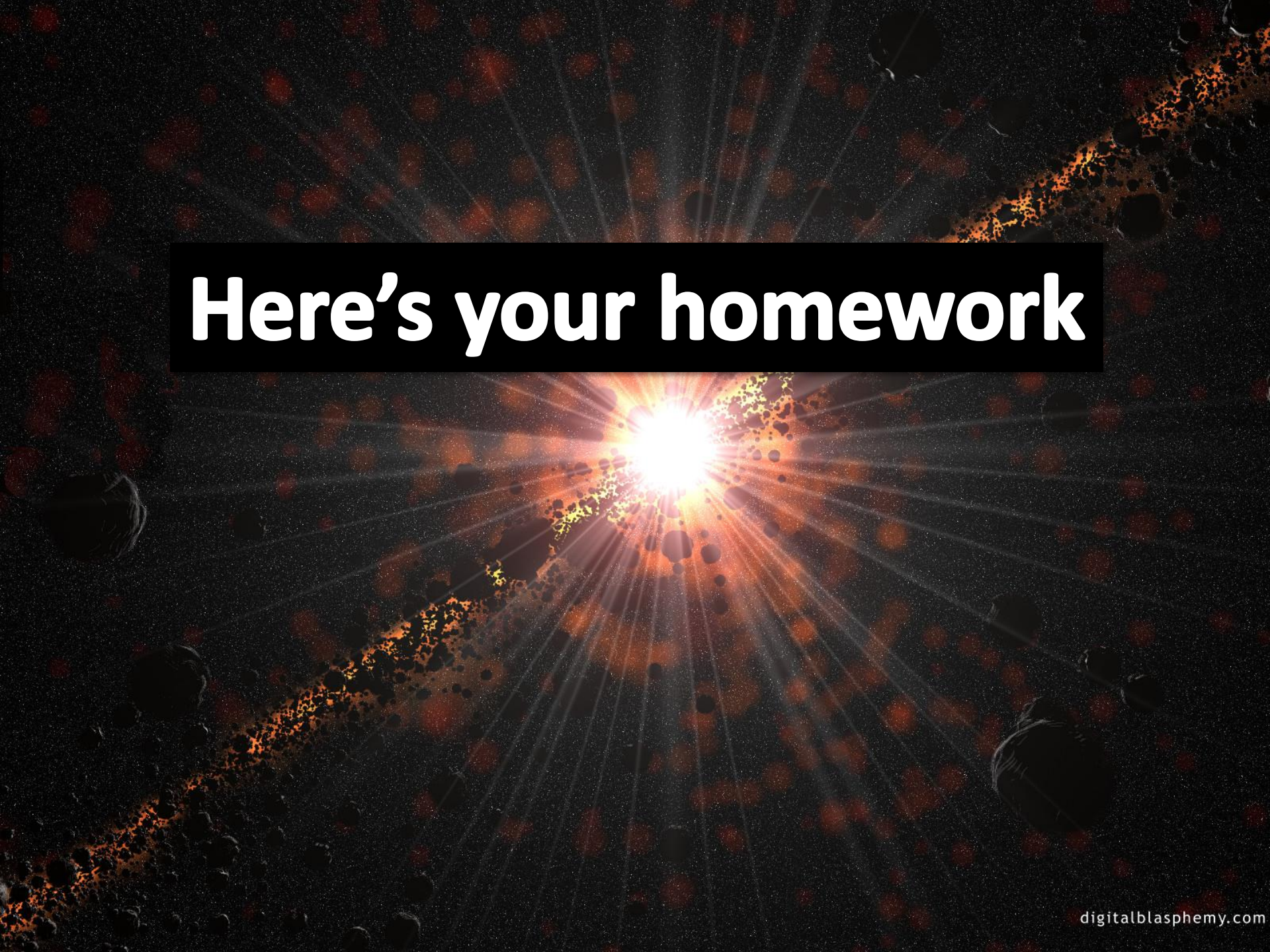
At $(-1;2)$... $f'(x) = -0.5$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{1}{2}(x - (-1))$$

$$y - 2 = -\frac{1}{2}x - \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$



The background is a dark, textured space filled with numerous small, glowing orange and red speckles, resembling distant stars or dust. A bright, intense white and yellow light source, possibly a star or a nebula, is positioned in the center, radiating outwards in all directions with sharp, thin lines of light. A prominent diagonal band of orange and black speckles runs from the bottom left towards the top right, passing through the central light source.

Here's your homework

For each of the functions below, determine the equations of the tangent.

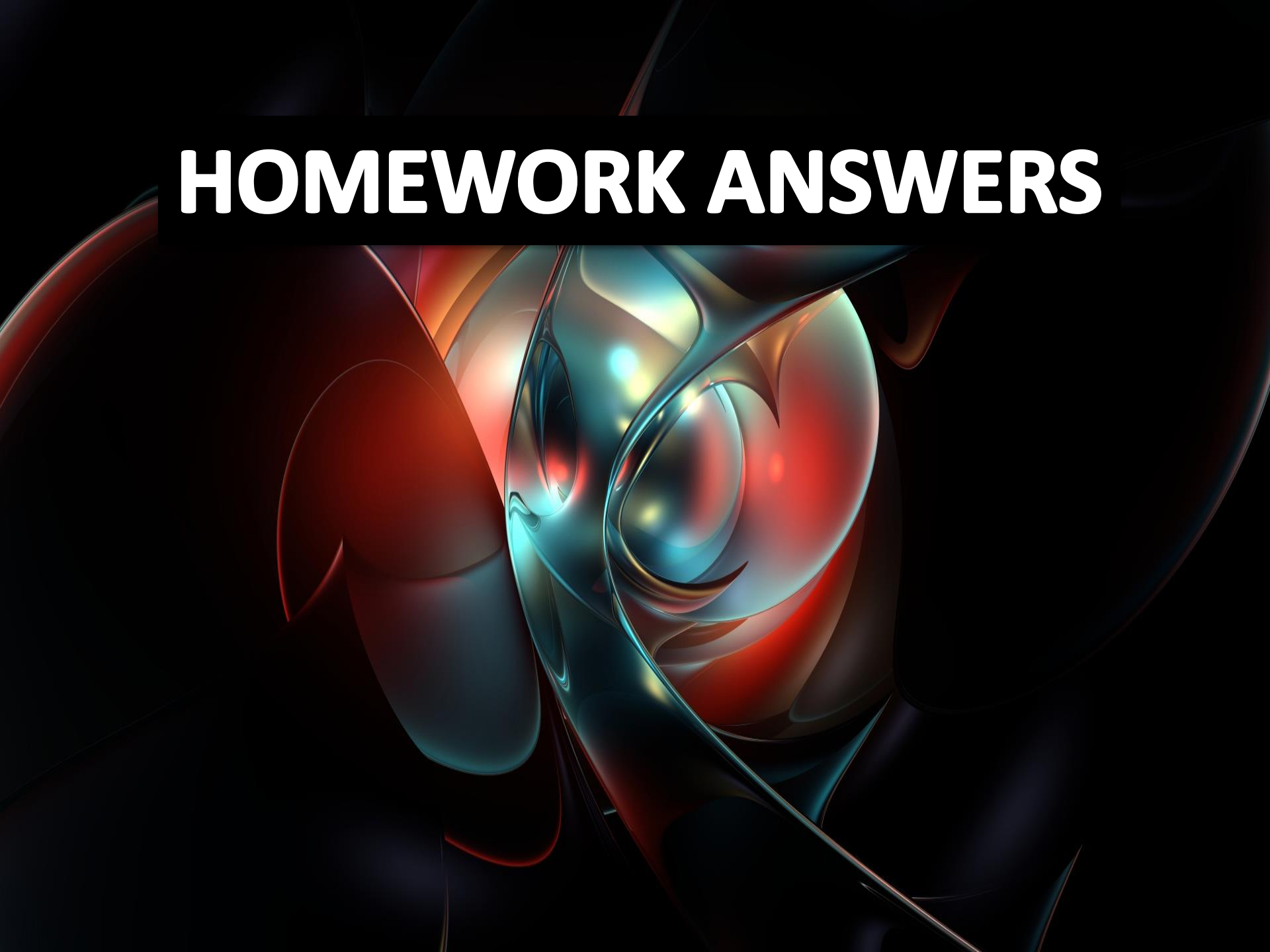
$a) f(x) = x^2 + 3x + 1 \dots \text{at } x = 0 \text{ \& } x = 4$

$b) f(x) = 2x^3 - 5x + 4 \dots \text{at } x = -1 \text{ \& } x = 1$

$c) f(x) = \tan x \dots \text{at } x = \frac{\pi}{4}$

$d) f(x) = 3 - x \dots \text{at } x = -1 \text{ \& } x = 0$

HOMEWORK ANSWERS



a) for $x = 0$

$$y = x^2 + 3x + 1$$
$$y = (0)^2 + 3(0) + 1$$
$$y = 1$$
$$(0; 1)$$

$$f'(x) = 2x + 3$$
$$f'(x) = 2(0) + 3$$
$$f'(x) = 3$$
$$m = 3$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 3(x - 0)$$
$$y - 1 = 3x$$
$$y = 3x + 1$$

for $x = 4 \dots$

$$y = x^2 + 3x + 1$$
$$y = (4)^2 + 3(4) + 1$$
$$y = 29$$

$$f'(x) = 2x + 3$$
$$f'(x) = 2(4) + 3$$
$$f'(x) = 11$$
$$m = 11$$

$$y - y_1 = m(x - x_1)$$
$$y - 29 = 11(x - 4)$$
$$y - 29 = 11x - 44$$
$$y = 11x - 15$$

b) *for* $x = 1$

$$f(x) = 2x^3 - 5x + 4$$

$$f(-1) = 2(-1)^3 - 5(-1) + 4$$

$$f(-1) = 7$$

$$(-1; 7)$$

$$f'(x) = 6x^2 - 5$$

$$f'(-1) = 6(-1)^2 - 5$$

$$f'(-1) = 1$$

$$m = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 1(x - (-1))$$

$$y - 7 = x + 1$$

$$y = x + 8$$

for $x = 1$

$$f(x) = 2x^3 - 5x + 4$$

$$f(1) = 2(1)^3 - 5(1) + 4$$

$$f(1) = 1$$

$$(1; 1)$$

$$f'(x) = 6x^2 - 5$$

$$f'(1) = 6(1)^2 - 5$$

$$f'(1) = 1$$

$$m = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x - (-1))$$

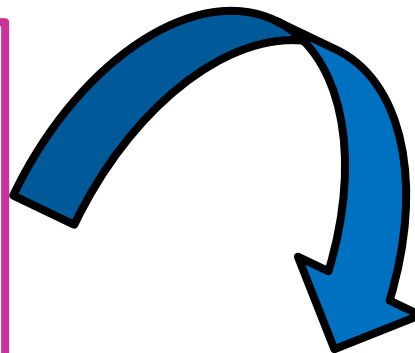
$$y - 1 = x - 1$$

$$y = x$$

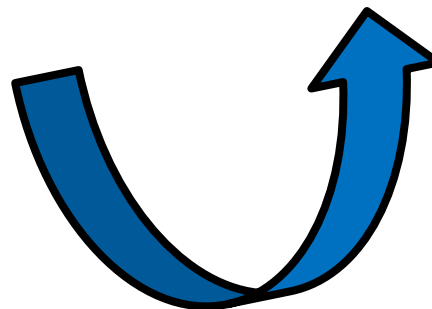
c)

$$\begin{aligned}f(x) &= \tan x \\f\left(\frac{\pi}{4}\right) &= \tan\left(\frac{\pi}{4}\right) \\&= 1 \\&\left(\frac{\pi}{4}; 1\right)\end{aligned}$$

$$\begin{aligned}f'(x) &= \sec^2 x \\&= \sec^2 \frac{\pi}{4} \\&= \left(\frac{1}{\cos \frac{\pi}{4}}\right)^2 \\f'(x) &= 2 \\m &= 2\end{aligned}$$



$$\begin{aligned}y - y_1 &= m(x - x_1) \\y - 1 &= 2\left(x - \frac{\pi}{4}\right) \\y - 1 &= 2x - \frac{2\pi}{4} \\y &= 2x - \frac{\pi}{2} + 1\end{aligned}$$



d) *for* $x = -2$

$$f(x) = 3 - x$$

$$f(-2) = 3 - (-2)$$

$$f(-2) = 5$$

$$f'(x) = -1$$

$$m = -1$$

$$\mathbf{y - y_1 = m(x - x_1)}$$

$$y - 5 = -1(x + 2)$$

$$y - 5 = -x - 2$$

$$y = -x + 3$$

for $x = 0$

$$f(x) = 3 - x$$

$$f(0) = 3 - (0)$$

$$f(0) = 3$$

$$f'(x) = -1$$

$$m = -1$$

$$\mathbf{y - y_1 = m(x - x_1)}$$

$$y - 3 = -1(x - 0)$$

$$y - 3 = -x$$

$$y = -x + 3$$



...Well that's about it for now...