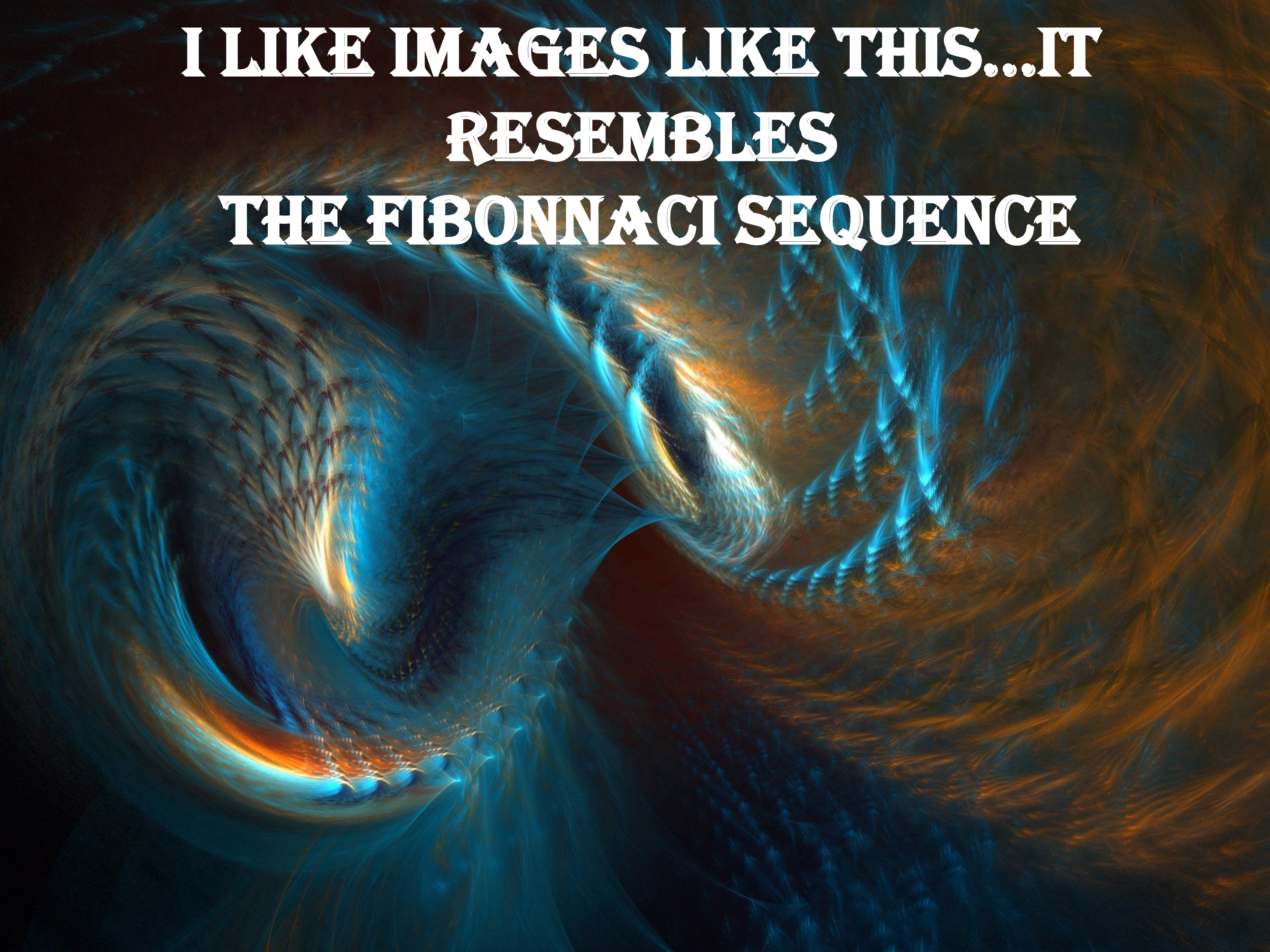
Limits and Continuity


I LIKE IMAGES LIKE THIS…IT RESEMBLES
THE FIBONNACI SEQUENCE

THIS ALSO LOOKS REALLY COOL…

...OKAY LETS GET STARTED...

LIMIT

So lets define a limit…

A **LIMIT** is the number that a function approaches as the **INDEPENDENT** variable of a function approaches a given value…

LIMIT

For example…

Given the function $f(x) = 3x$…

You could say "the limit of $f(x)$ as x approaches 2….is…. 6".

CONTINUITY

A FUNCTION CAN EITHER BE...

CONTINUOUS

DISCONTINUOUS

One way to test for the continuity of a function is to see whether the graph of a function can be traced with a pen...without lifting the pen from the paper

CONTINUITY

A function is a rule that assigns each element x from a set known as the **DOMAIN**…a single element y from a set known as the **RANGE**.

CONTINUITY

For example…

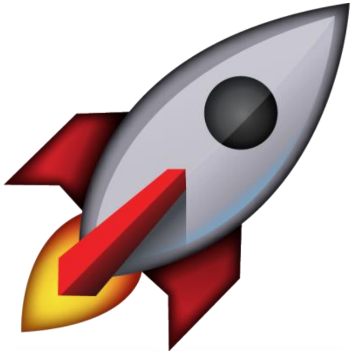
$$y = x^2 + 2$$

Assigns the value…

$$y = 3 \; to \; x = 1$$

&

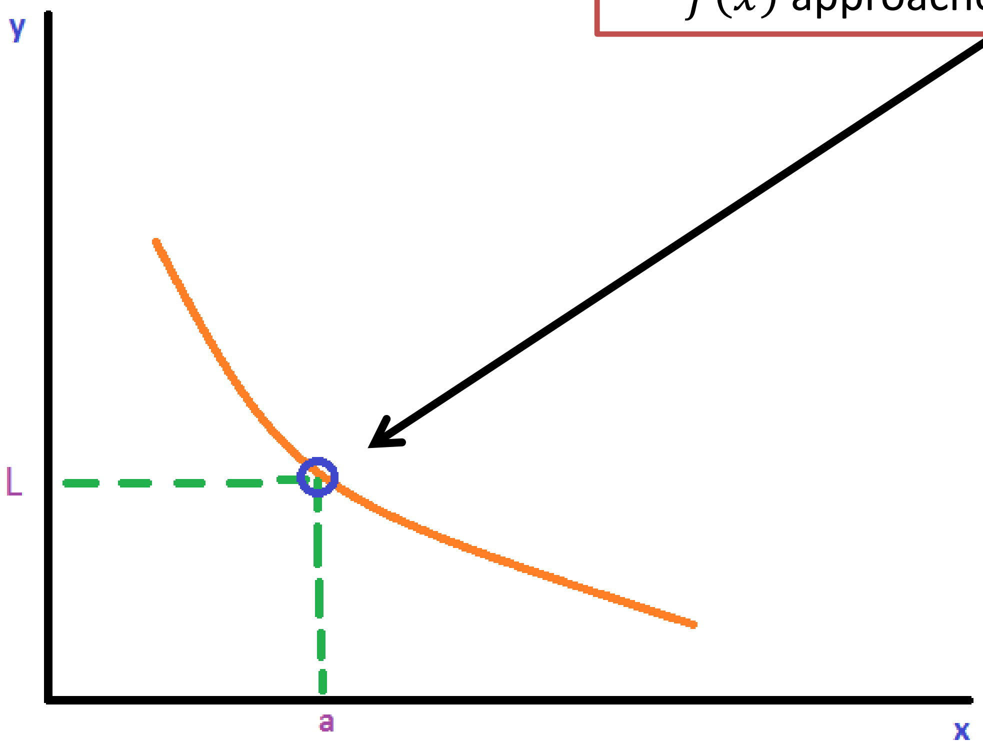
$$y = 6 \; to \; x = 2$$



The idea behind limits is to understand what the function is "approaching" when x "approaches" a specific value.

LEtS tURN tHIS INtO A GRAPH...

When x approaches the value "a" in the X-axis, the function $f(x)$ approaches L on the Y-axis



CONTINUITY

Lets consider $f(x) = x^2$

Focus on the point (1;1)

When x approaches 1, $f(x)$ approaches 1.
When this happens we can say:

$$\lim_{x \to 1} x^2 = 1$$

CONTINUITY

So now you might ask…why is this useful?

Why would you need to know what the function is approaching?

You already know the function equals 1 when x equals 1.

CONTINUITY

Lets consider the following…

$$\boldsymbol{f(x) = \begin{cases} x^2 & if\ x \neq 1 \\ 0 & if\ x = 1 \end{cases}}$$

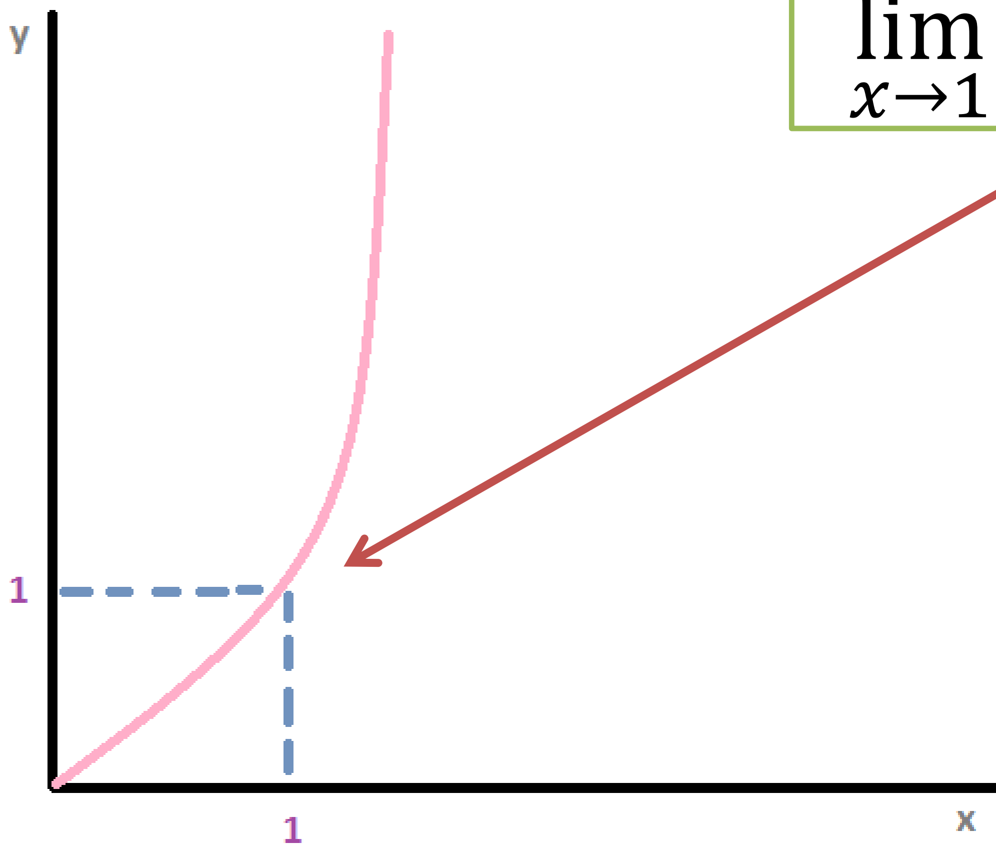
Don't let this intimidate you!!!

This only means that this function equals x^2 when x is anything other than 1 & equals 0 when x equals 1.

Never the less…lets graph it out…

CHECK OUT THE GRAPH…

$$\lim_{x\to 1} f(x) = 1$$

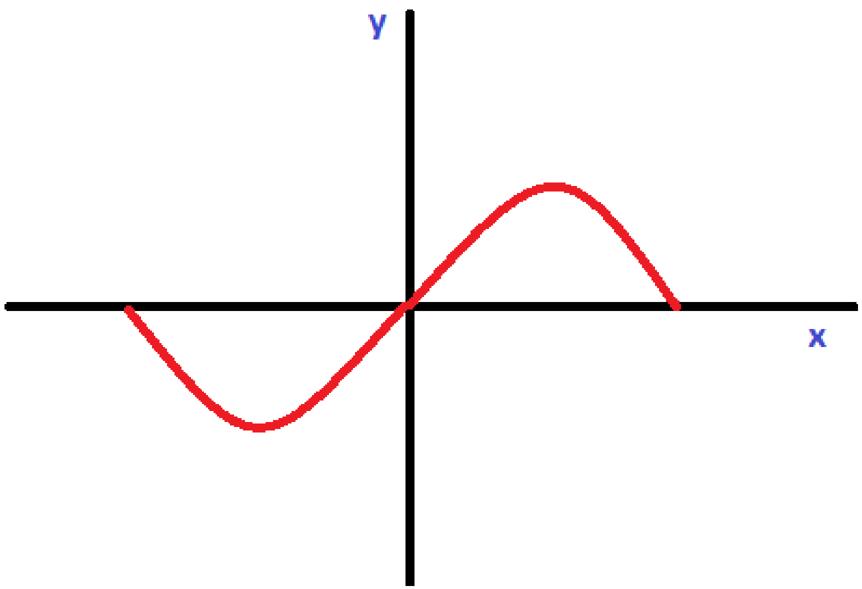


CONTINUITY

Limits and Continuity are related.

Remember…a function is continuous when you can graph it without lifting your pen from the paper…

Here is a nice example…



Continuity

Now…what would a discontinuous function look like?

A function is discontinuous when it has any “gap”

There are 3 types of discontinuity …

DISCONTINUITY

1. Asymptote discontinuity

Asymptotes occur when a function approaches ∞ at a specific value of x or y.

If a function has values on both sides of an asymptote, then it cannot be connected.

It therefore must have a discontinuity at the asymptote.

Discontinuity

We look for asymptotes at points where the denominator is zero.

When the denominator gets close to zero, and becomes small, it makes the value of the function very large.

$$\frac{5}{0.1} = 50$$

$$\frac{5}{0.01} = 500$$

$$\frac{5}{0.00001} = 500\,000$$

Therefore…the closer to zero the denominator is…the larger the value of the function.

$$\lim_{x \to 0} \frac{1}{x} = \infty$$

DISCONTINUITY

2. Point discontinuity

When a function is defined specifically for an isolated x value. If we change our function slightly too…

$$\boldsymbol{f(x) = \begin{cases} 9 & x = 3 \\ x^2 & all\,real\,x\,values \end{cases}}$$

It becomes continuous.
We defined the value at $f(3)$ to be the value of the function $f(x) = x^2 at\ x = 3$

DISCONTINUITY

3. Jump discontinuity

Just as we can define a function at a specific point, we can also define a function in specific regions.

Consider…

$$\boldsymbol{f(x) = \begin{cases} x^2; x \leq 1 \\ 2 - x; x > 1 \end{cases}}$$

DISCONTINUITY

Lets work on an example…

Estimate the value of the following limit:

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$$

Lets choose values of x that get closer and closer to x = 2

DISCONTINUITY

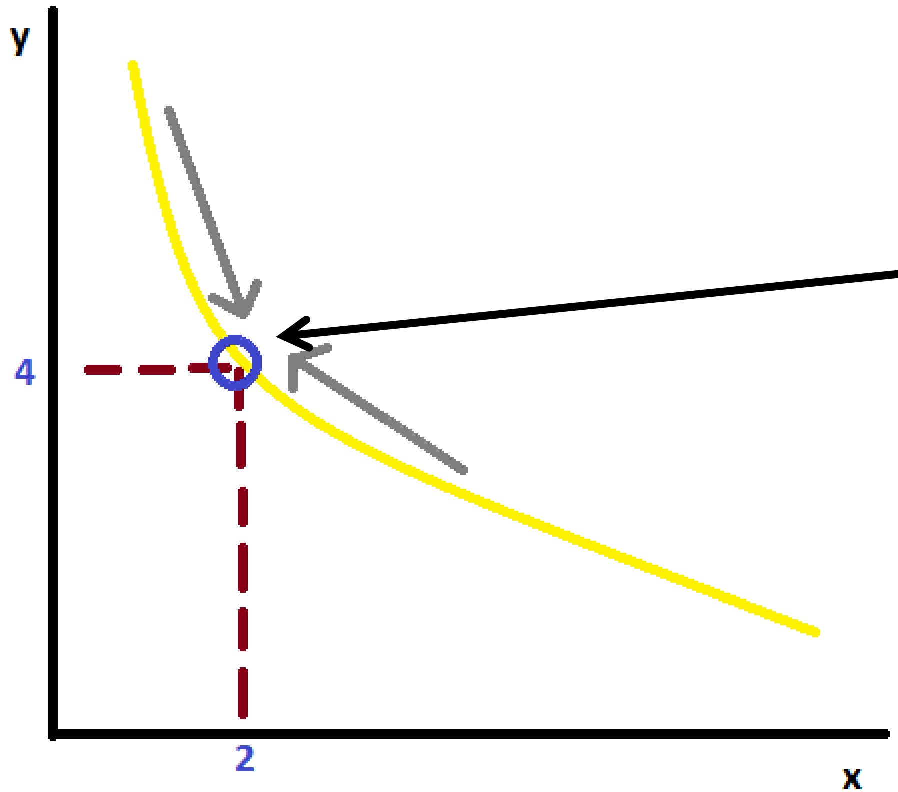
x	$f(x)$	x	$f(x)$
2.5	3.4	1.5	5.0
2.1	3.85	1.9	4.15
2.01	3.98	1.99	4.01
2.001	3.99	1.999	4.0015

From this table…
It appears that the function is going to 4…as x approaches 2

$$\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x} = 4$$

DISCONTINUITY

Lets think a little bit more about what's going on here…



Look at the blue circle.
The arrows show we are moving towards this point as we plug in the values of x.

Discontinuity

When we are calculating limits…what we are really asking is…

What y value is our graph approaching…as we move in towards our graph.

We are NOT asking what y value the graph takes at the point.

...so that's it from me...

...here are some nice images to end of with...=)