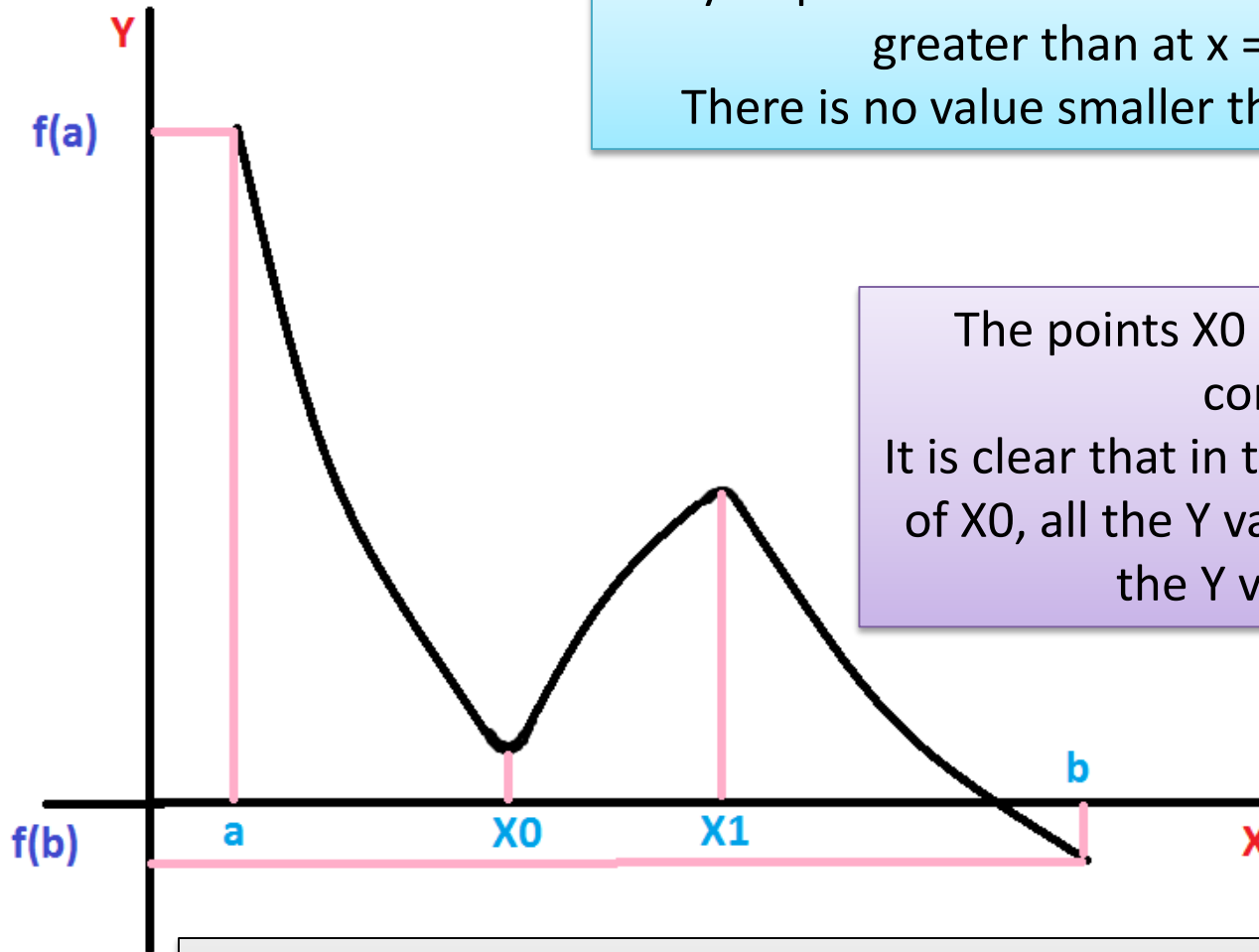




MAXIMA & MINIMA & APPLICATIONS

By inspection we see that there is no Y value greater than at $x = a$ [$f(a)$]
There is no value smaller than at $x = b$ [$f(b)$]

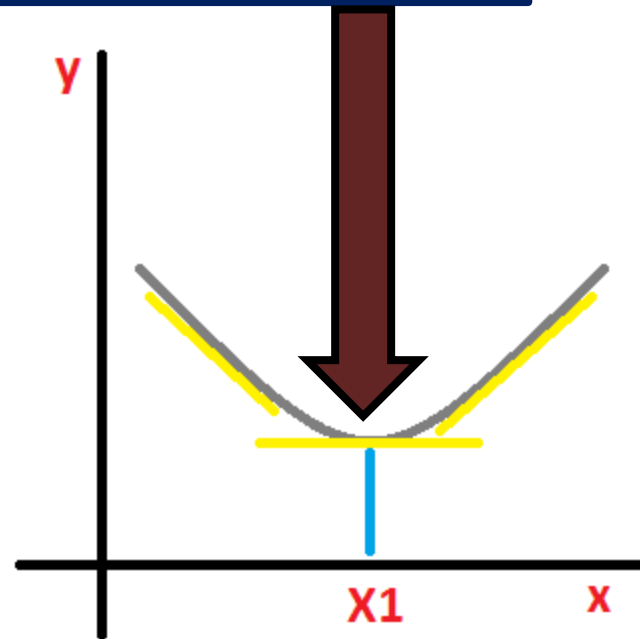
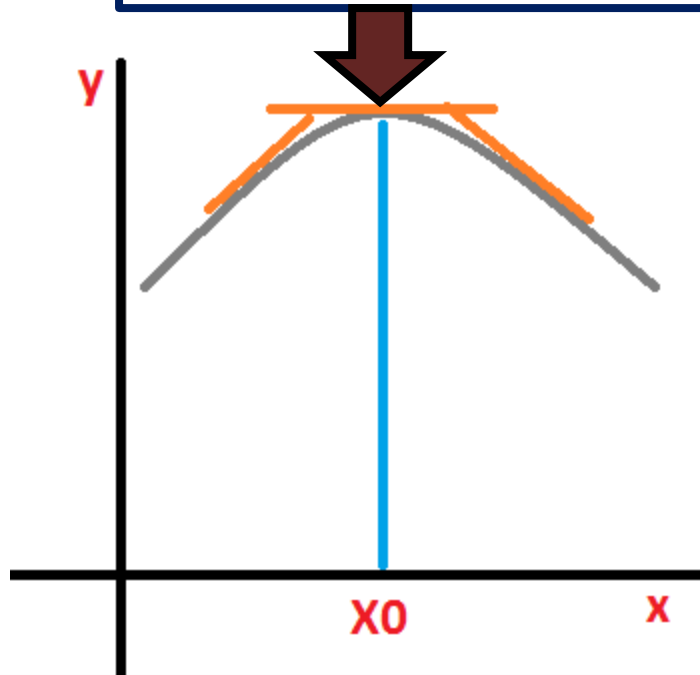


The points $X0$ and $X1$ also merit a comment.
It is clear that in the near neighborhood of $X0$, all the Y values are greater than the Y values at $X0$.

So $f(x)$ has a **GLOBAL MAXIMUM** at $x = a$ & a **GLOBAL MINIMUM** at $x = b$
It also has a **LOCAL MAXIMUM** at $x = X1$ & a **LOCAL MINIMUM** at $x = X0$

A stationary point on a curve is one at which the derivative has a zero value

STATIONARY POINTS



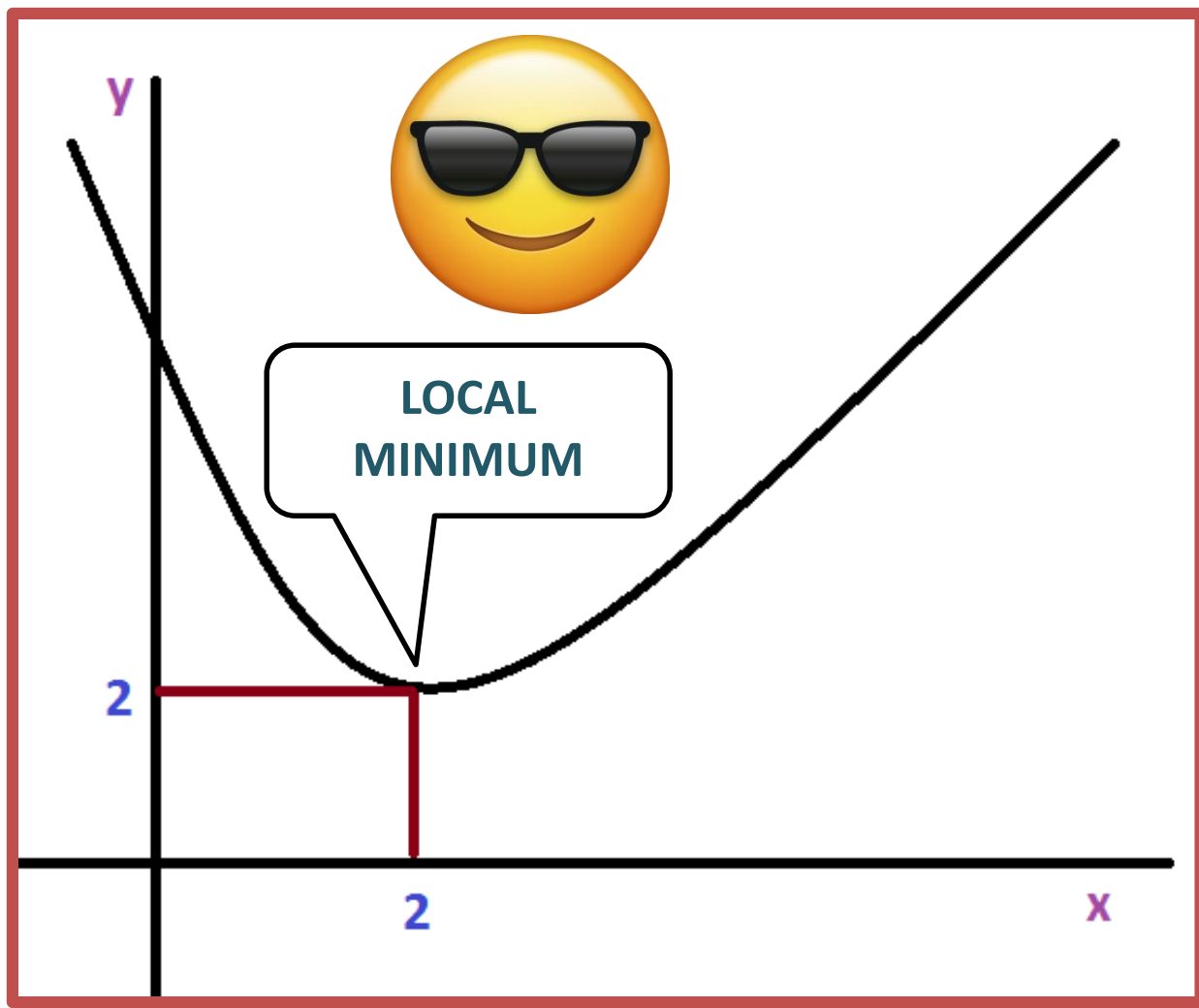
Points on the curve $y = f(x)$ at which $\frac{df}{dx} = 0$ are called **STATIONARY POINTS** of the function

Example:

Sketch the curve

$$y = (x - 2)^2 + 2$$

& locate the stationary points
on the curve.



At a stationary point...

$$\frac{df}{dx} = 0$$

We have

$$y = (x - 2)^2 + 2$$

$$\text{So } x = 2$$

The function has 1 stationary point at $x = 2$, $y = 2$.

This point is also called a **LOCAL MINIMUM**

Example:

Find the stationary points:

$$a) \ f(x) = 3x^2 + 2x - 9$$

$$b) \ f(x) = x^3 - 6x^2 + 9x - 2$$

a)

$$f(x) = 3x^2 + 2x - 9$$
$$f'(x) = 6x + 2$$

The stationary points are found by solving the equation ...

$$f'(x) = 0$$
$$6x + 2 = 0$$
$$x = -\frac{1}{3}$$

Sub x in $f(x)$

$$f(x) = 3x^2 + 2x - 9$$
$$= 3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) - 9$$
$$= -\frac{28}{3}$$

b)

$$f(x) = x^3 - 6x^2 + 9x - 2$$

$$f'(x) = 3x^2 - 12x + 9$$

$$\begin{aligned} 3x^2 - 12x + 9 &= 0 \\ &= 3(x^2 - 4x + 3) \\ &= 3(x - 3)(x - 1) \\ x &= 3 \text{ \& } x = 1 \end{aligned}$$

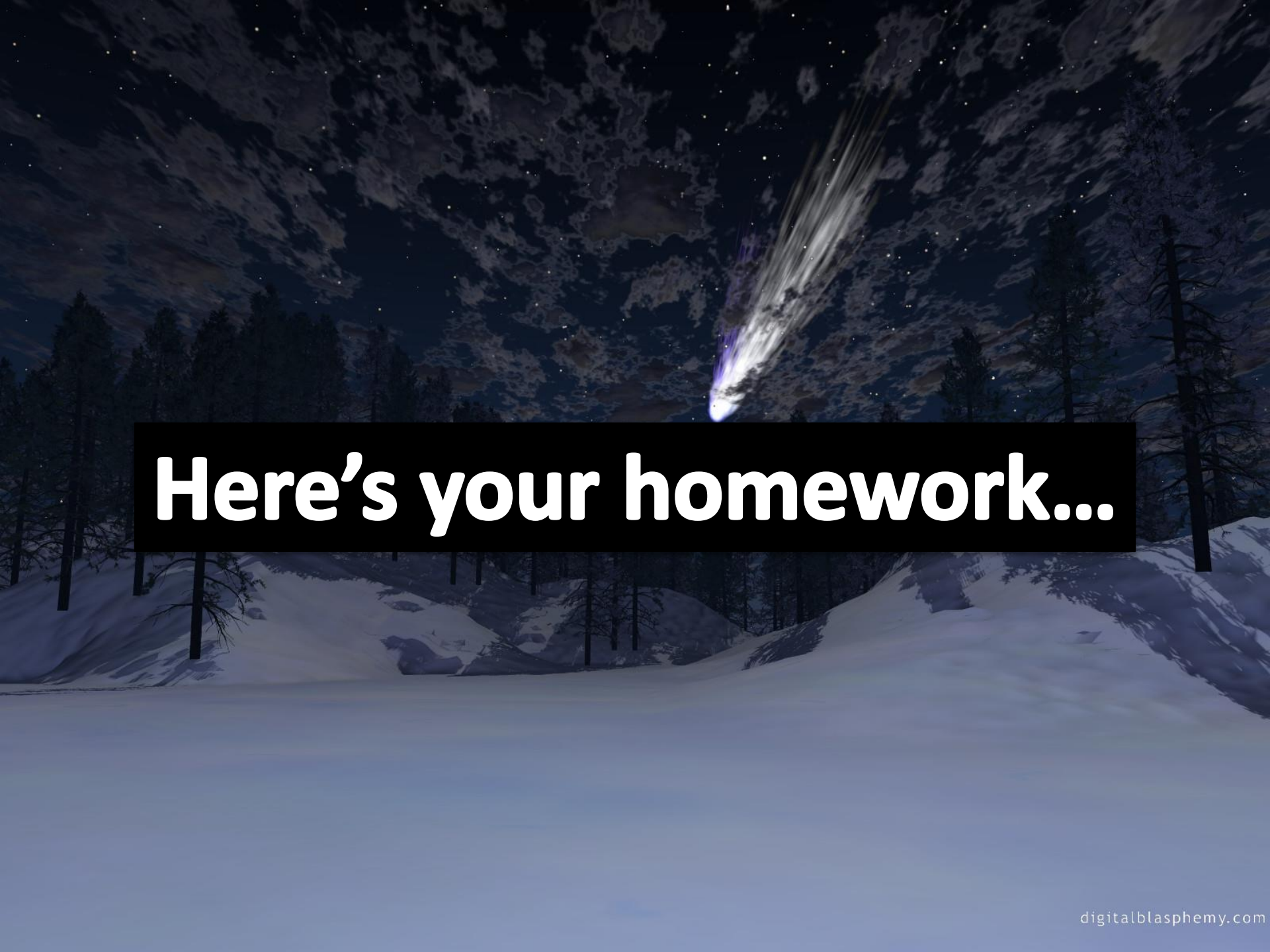
Sub $x = 3$ and $x = 1$ in $f(x)$

$$3(3)^2 - 12(3) + 9$$

&

$$3(1)^2 - 12(1) + 9$$

stationary points are (3; 2) & (1; 2)

A night scene of a snowy mountain landscape. In the foreground, there is a smooth, snow-covered slope. In the middle ground, a line of dark evergreen trees stands against the night sky. The sky is filled with stars and wispy clouds. A bright meteor streaks diagonally across the upper right portion of the sky, leaving a long, glowing trail. A black rectangular box with white text is centered over the middle of the image.

Here's your homework...

Exercise 1:



$$1. f(x) = -u^2 - 6u + 16$$

$$2. f(x) = 3x^2 - 4x + 7$$

$$3. f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$$

$$4. f(x) = x^3 - 6x^2 - 15x + 16$$

Find the stationary point of the function

$$y = x^2 - 2x + 3$$

& hence determine the nature of this point.

Determine the nature of the stationary points of the following functions...

$$y = 16 - 6u - u^2$$

$$y = 3x^2 - 4x + 7$$

Find the stationary points of the function

$$y = 2x^3 - 9x^2 + 12x - 3$$

& determine their nature

Find the maximum and minimum values of the function

$$y = 4x^3 + 3x^2 - 6x$$

On the interval [-2;1]

A wide-angle photograph of a beach at sunset or sunrise. In the foreground, the wet sand is dark and reflective, showing ripples and some seaweed. In the middle ground, several large, dark, jagged rock formations stand prominently. The sky is a vibrant mix of colors, with deep blues and purples on the left, transitioning into bright oranges and yellows near the horizon where the sun is setting. Large, fluffy clouds are illuminated from below, creating a dramatic silhouette effect. The overall mood is serene yet powerful.

APPLICATIONS

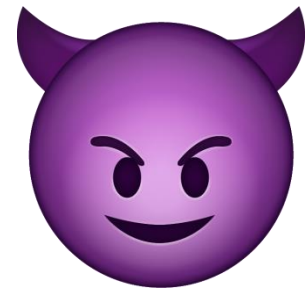
Applied Maxima & Minima Problems

The process of finding maximum and minimum values is called **OPTIMIZATION**.

We are trying to do things like maximize the profit in a company, or minimize the costs, or find the least amount of material to make a product.



**SHOW ME
THE
MONEY!!!**



Example 1:

The daily profit of an oil refinery is given by:

$$P = 8x - 0.02x^2$$

x = number of barrels of oil refined

How many barrels will give the maximum profit?

What is the maximum profit?

The profit is a maximum at $\frac{dp}{dx} = 0$

$$\frac{dp}{dx} = -0.04x + 8$$

$$-0.04x + 8 = 0$$

$$-0.04x = -8$$

$$x = \frac{-8}{-0.04}$$

$$x = 200$$

Is it a maximum?

$$\frac{d^2P}{dx^2} = -0.04 \dots$$

< 0 thus its a maximum

When $x = 200 \dots$

$$P = 8(200) - 0.02(200)^2$$

$$P = 800$$

*If the company refines
200 barrels a day ...*

*a maximum profit
of 800 is reached*

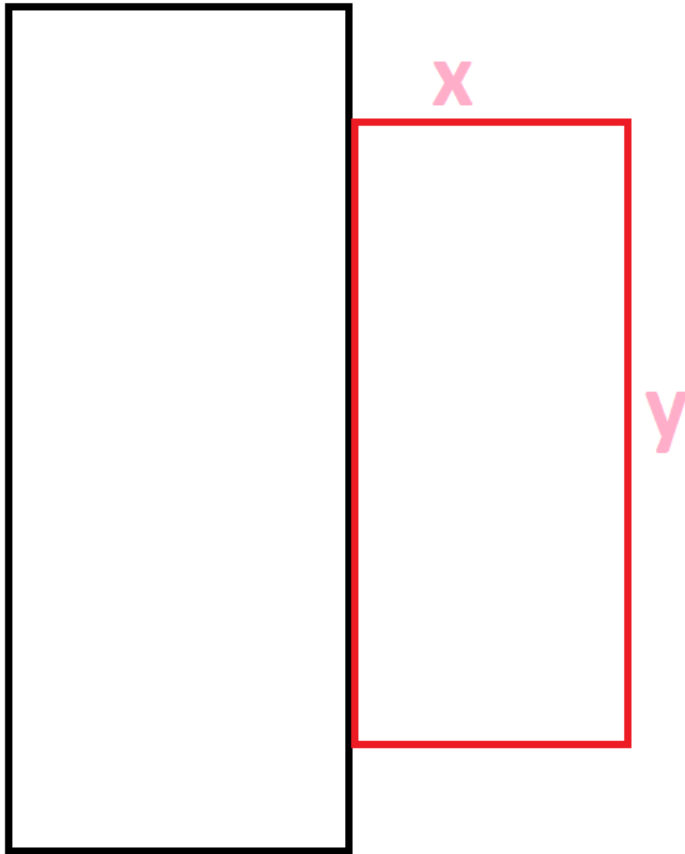
Example 2:

A rectangular storage area is to be constructed along the side of a tall building.

A security fence is required along the remaining 3 sides of the area.

What is the maximum area that can be enclosed with 800m of fencing?

HERE'S WHAT
IT LOOKS
LIKE...



$$\text{Area} = xy$$

$$2x + y = 800$$

$$y = 800 - 2x$$

$$\text{Area} = xy$$

$$= x(800 - 2x)$$

$$\text{Area} = 800x - 2x^2$$

To maximise the area ...

$$\text{find when } \frac{dA}{dx} = 0$$

$$\frac{dA}{dx} = 800 - 4x$$

$$800 - 4x = 0$$

$$x = \frac{-800}{-4}$$

$$x = 200$$

Substitute $x = 200$ in y

$$\begin{aligned} 800 - 2(200) \\ = 400 \end{aligned}$$

Is it a maximum?

$$\frac{d^2 A}{dx^2} = -4 \dots$$

< 0 ... *Therefore maximum*

Maximum area occurs when

$$x = 200 \text{ \& } y = 400$$

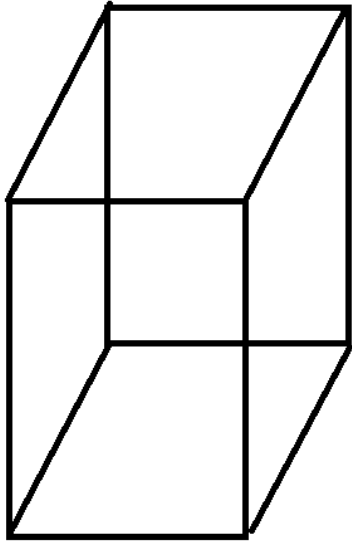
$$\begin{aligned} \text{Area} &= 200 \times 400 \\ &= 80\,000 \text{m}^2 = 8 \text{ ha} \end{aligned}$$

Example 3:

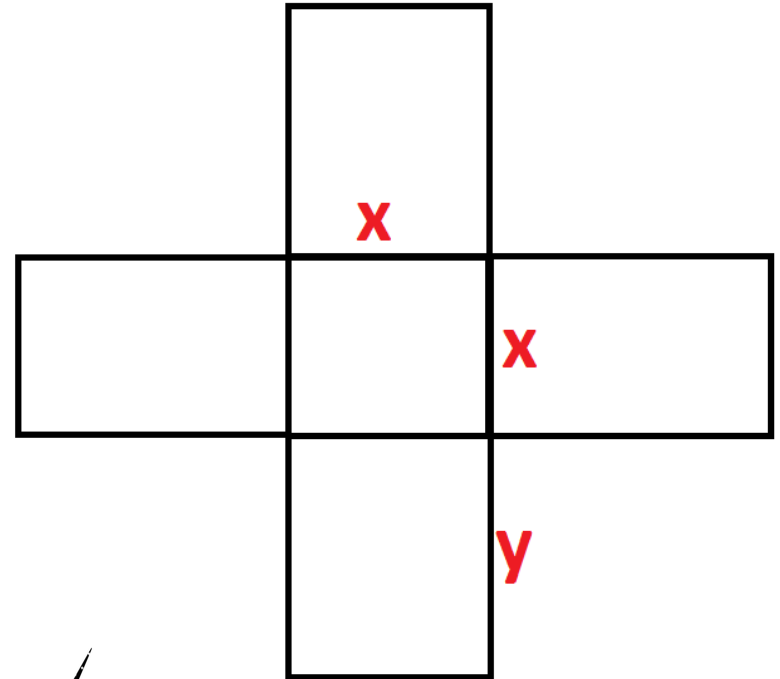
A box with a square base has no top.

If 64cm^2 of material is used, what is the maximum possible volume for the box?





OPEN IT UP...



$$\text{Volume} = x^2y$$

Surface area of the box is 64cm^2

Area of the base of the box is x^2

& area of each side is xy

*Area of the base + area of the 4 sides
is given by ...*

$$x^2 + 4xy = 64\text{cm}^2$$

$$4xy = 64 - x^2$$

$$y = \frac{64 - x^2}{4x}$$

$$y = \frac{16}{x} - \frac{x}{4}$$



*volume can be
re-written as*

$$\begin{aligned} V &= x^2 y \\ &= x^2 \left(\frac{16}{x} - \frac{x}{4} \right) \\ &= 16x - \frac{x^3}{4} \end{aligned}$$

$$\begin{aligned} \frac{dV}{dx} &= 16 - \frac{3}{4}x^2 \\ 16 - \frac{3}{4}x^2 &= 0 \\ x &= 4.62 \end{aligned}$$

Substitute $x = 4.62$ in y ...

$$\begin{aligned} y &= \frac{16}{x} - \frac{x}{4} \\ &= \frac{16}{4.62} - \frac{4.62}{4} \\ &= 2.31 \end{aligned}$$

is it a maximum?

$$\frac{d^2V}{dx^2} = -\frac{3}{2}x \dots < 0 \dots \text{Maximum}$$

$$\begin{aligned} \text{Maximum Volume} &= x^2 y \\ &= (4.62)^2 (2.31) \\ &= 49.3 \text{ cm}^3 \end{aligned}$$

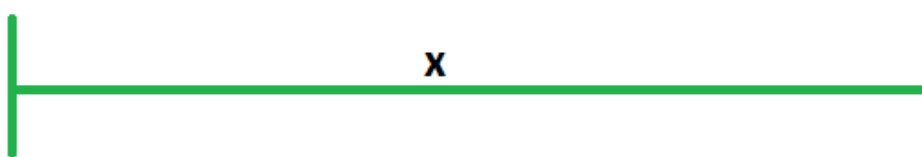
$$\begin{aligned} \text{Area} &= x^2 + 4xy \\ &= (4.62)^2 + 4(4.62)(2.31) \\ &= 64 \end{aligned}$$

Example 4:

A rectangular lot is bounded at the back by a river.

No fence is need along the river, and there has to be a 24ft opening at the front.

If the fence along the front costs R1.50 per foot, & along the sides R1.00 per foot, find the dimensions of the largest lot which can be fenced for R300.



Total cost:

$$300 = 2y + 1.5(x - 24)$$

$$y = 168 - 0.75x$$

Area = xy

$$= x(168 - 0.75x)$$

$$= 168x - 0.75x^2$$

$$\frac{dA}{dx} = 168 - 1.5x$$

$$-1.5x + 168 = 0$$

$$-1.5x = -168$$

$$x = \frac{-168}{-1.5}$$

$$x = 112ft$$

$$y = 168 - 0.75(112)$$

$$y = 84ft$$

dimensions = 84ft × 112ft