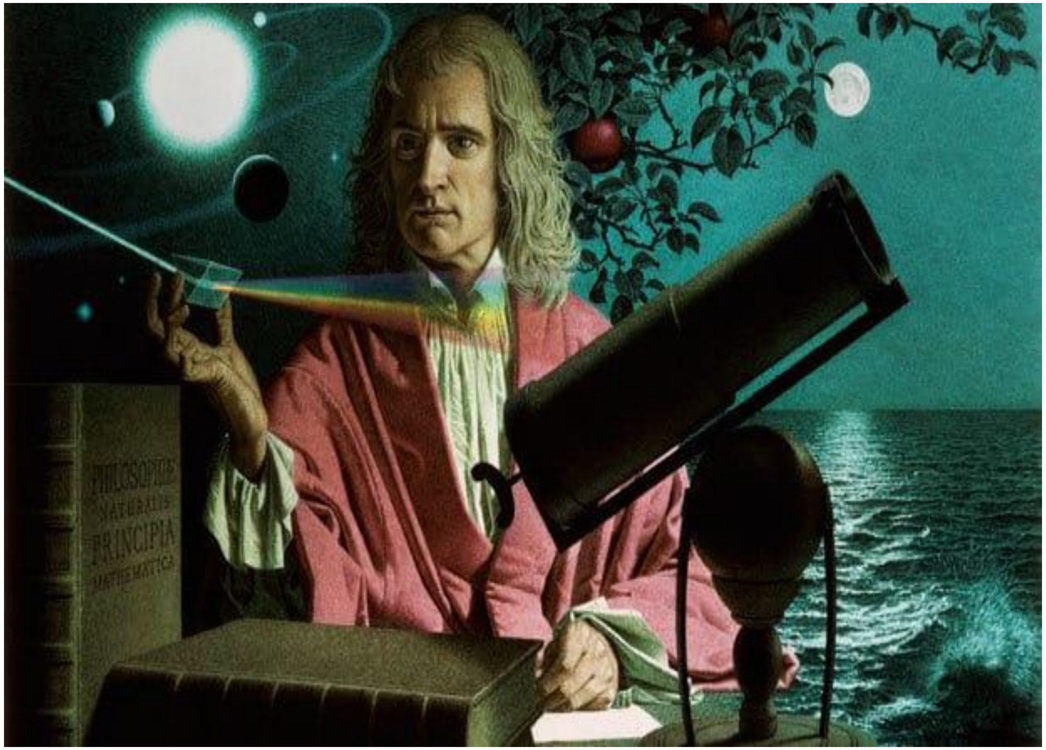
NEWTON’S LAW OF COOLING

Here’s our Good friend Newton...

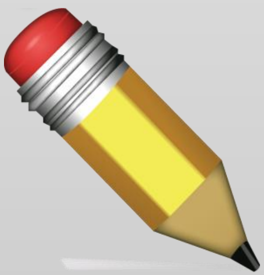


Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).

The law of cooling states that the rate of cooling of an object is proportional to the difference between the object and its surroundings.

Let T(t) be the temperature of an object at time t, and $\mathbf{T_s}$ be the **temperature of the surroundings.**

$\mathbf{T_s}$ **is assumed to be constant.**



$$\frac{dT}{dt} = k(T - T_s)$$

The rate that the temperature T of an object that is cooling is given by the initial value problem…

$$\frac{dT}{dt} = k(T - T_s) \ldots T(0) = 0$$

The solution of this initial value problem describing the objects temperature after time t...

Initial temperature

Proportionality constant

$$T(t) = T_s + (T_0 - T_s).e^{kt}$$

Temperature At time t

Temperature of Surroundings

Lets look at some examples

Example 1:

Suppose you cool a pot of soup in a 75° F room.

Right when you take the soup of the stove, you measure its temperature to be 220°F.

Suppose after 20 minutes the soup has cooled to 170°F.

a) What will the soup temperature be after 30 minutes?
b) Suppose you can eat the soup when its 130°F. How long will it take to cool to this temperature?

$$T_s = 75^oF \; \ldots Surroundings$$
$$T_0 = 220^oF \ldots Initial\ value$$

$$T(t) = T_s + (T_0 - T_s).\, e^{kt}$$

$$= 75 + (220 \; - 75).\, e^{kt}$$

$$T(t) = 75 + 145.\, e^{kt}$$

WE WRITE
AN EQUATION....

$$After\ 20\ Minutes\ the\ temperature\ is\ 170^{o}F\ \ldots$$
$$T(20) = 170$$

$$T(20) = 170 = 75 + 145.\, e^{k(20)}$$
$$75 + 145.\, e^{20k} = 170$$
$$145.\, e^{20k} = 170 - 75$$
$$145.\, e^{20k} = 95$$
$$e^{20k} = \frac{95}{145}$$
$$\ln e^{20k} = \ln(\frac{95}{145})$$
$$20k \ln e = \ln(\frac{95}{145})$$
$$k = \frac{\ln \frac{95}{145}}{20}$$
$$k = -0.0211$$

Lets Solve For k...

$$T(t) = 75 + 145.e^{-0.0211(t)}$$

$$T(30) = 75 + 145.e^{-0.0211(30)}$$

$$152^{o}F$$

Putting it all together...

$$T(t) = 75 + 145.e^{-0.0211(t)}$$

$$130 = 75 + 145.e^{-0.0211(t)}$$

$$145.e^{-0.0211(t)} = 55$$

$$e^{-0.0211t} = \frac{55}{145}$$

$$\ln e^{-0.0211(t)} = \ln \frac{55}{145}$$

$$t = \frac{\ln \frac{55}{145}}{-0.0211}$$

$$t = 46\, minutes$$

Answer the Question...

Example 2:

At 06:00 am, a detective finds a murdered body. Temperature of the body is 30^{o}C.

By 08:00am the temperature dropped to 25^{o}C. Surrounding air temperature is 20^{o}C.

Using the fact that when alive, the human body is 37^{o}C…find out when the murder was committed.

$$@\ 0.6{:}00\ am \ldots t = 0$$

$$T(0) = 30^{o}C$$

$$T(t) = T_s + (T_0 - T_s).\,e^{kt}$$

$$T(0) = 30 = 20 + (30 - 20).\,e^{kt}$$

We start off

With our equation…

&

The solve for

k…

$$T(2) = 20 + (30 - 20).\,e^{2k}$$

$$25 = 20 + 10e^{2k}$$

$$10e^{2k} = 25 - 20$$

$$10e^{2k} = 15$$

$$e^{2k} = \frac{15}{10}$$

$$\ln e^{2k} = \ln\frac{15}{10}$$

$$2k = \ln\frac{15}{10}$$

$$k = \frac{\ln\frac{15}{10}}{2}$$

$$k = -0.3466$$

$$37 = 20 + (30 - 20).\, e^{-0.3466(t)}$$

$$37 = 20 + 10.\, e^{-0.3466(t)}$$

$$17 = 10.\, e^{-0.3466(t)}$$

$$\frac{17}{10} = e^{-0.3466(t)}$$

$$\ln\frac{17}{10} = \ln e^{-0.3466(t)}$$

$$\ln\frac{17}{10} = -0.3466(t)$$

$$(t) = \frac{\ln\frac{17}{10}}{-0.3466}$$

$$t = -1.53$$

Answer the question...

$Murder\ was\ committed\ 1\ and\ a\ half\ hours\ before\ the\ body\ was\ found$

Example 3:

A plate is heated to 60^oC and then cools to 40^oC in 15 minutes.

Surrounding air temperature is 20^oC.

a) Find the temperature of the plate after 30 minutes.
b) Find the time taken for the plate to cool to 25^oC.

$$T(t) = T_s + (T_0 - T_s).e^{kt}$$
$$T(15) = T_s + (T_0 - T_s).e^{kt}$$
$$40 = 20 + (60 - 20).e^{15k}$$
$$40 = 20 + 40.e^{15k}$$
$$20 = 40.e^{15k}$$
$$\frac{20}{40} = e^{15k}$$
$$\ln\frac{20}{40} = \ln e^{15k}$$
$$\ln\frac{20}{40} = 15k$$
$$k = \frac{\ln\frac{20}{40}}{15}$$
$$k = -0.0462$$

$$T(t) = T_s + (T_0 - T_s).e^{kt}$$
$$= 20 + (60 - 20).e^{-0.0462(t)}$$
$$= 20 + 40.e^{-0.0462(t)}$$
$$-\frac{20}{40} = e^{-0.0462(t)}$$
$$\ln -\frac{20}{40} = \ln e^{-0.0462(t)}$$
$$\ln -\frac{20}{40} = -0.0462(t)$$
$$t = \frac{\ln -\frac{20}{40}}{-0.0462}$$
$$t = 15$$

$$T = 20 + (60 - 20).e^{-0.0462(30)}$$
$$T = 30^oC$$

Example 4:

The temperature of a body dropped from $200^{o}C$ to $100^{o}C$ for the first hour.

Determine how many degrees the body cooled in 1 more hour if the environment temperature is $0^{o}C$.

Example 5:

A cheesecake is taken out of the oven with a temperature of 165^{o}F and placed into a 35^{o}F refrigerator.

After 10 minutes the cheesecake is cooled to 150^{o}F.

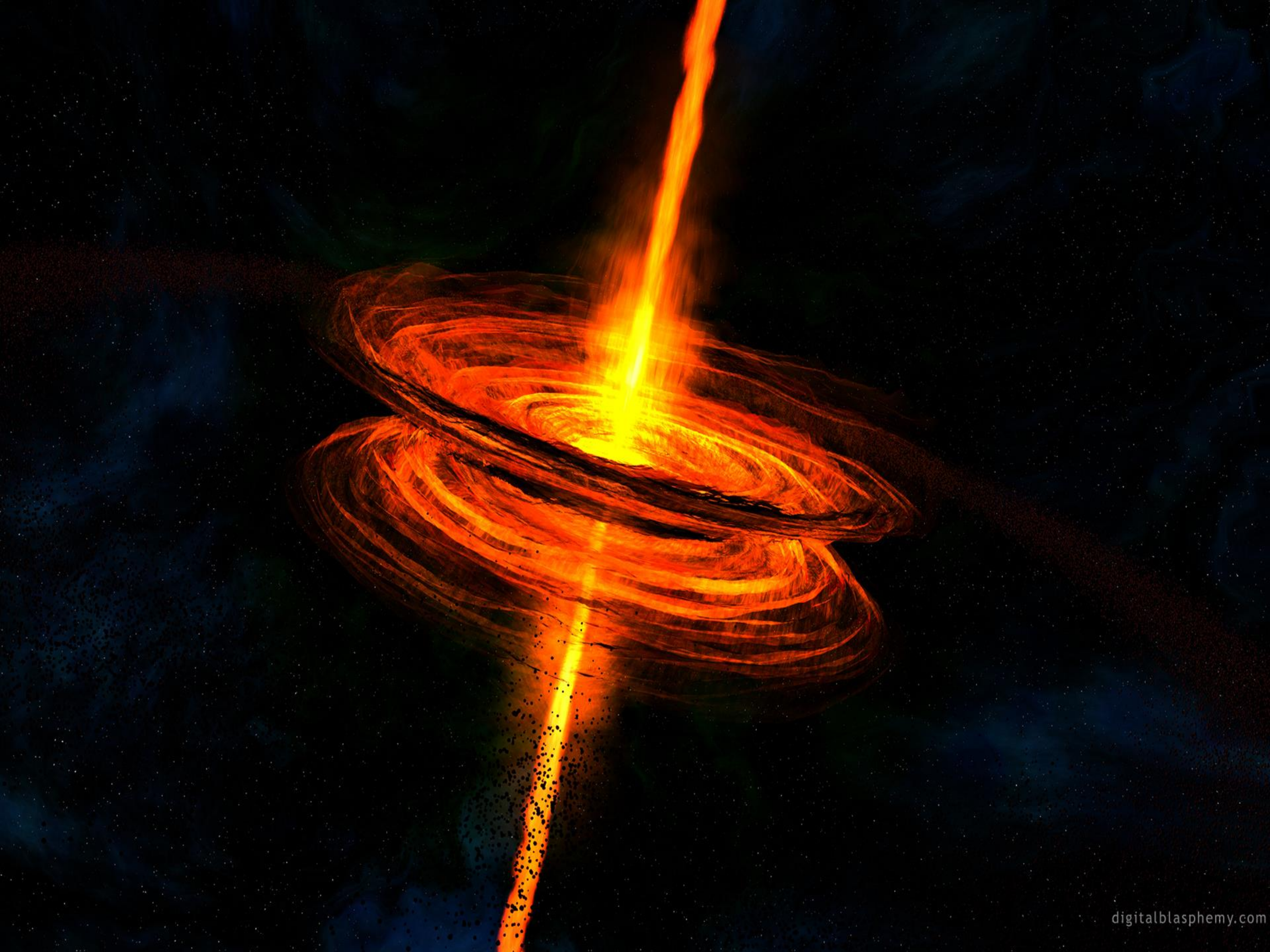
If we must wait till the cheesecake has cooled to 70^{o}F before we eat it…how long will we have to wait?

Example 6:

A pitcher of water at 40^oF is placed in a 70^oF room.

One hour later, the temperature has risen to 45^oF.

How long will it take for the temperature to be 60^oF?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