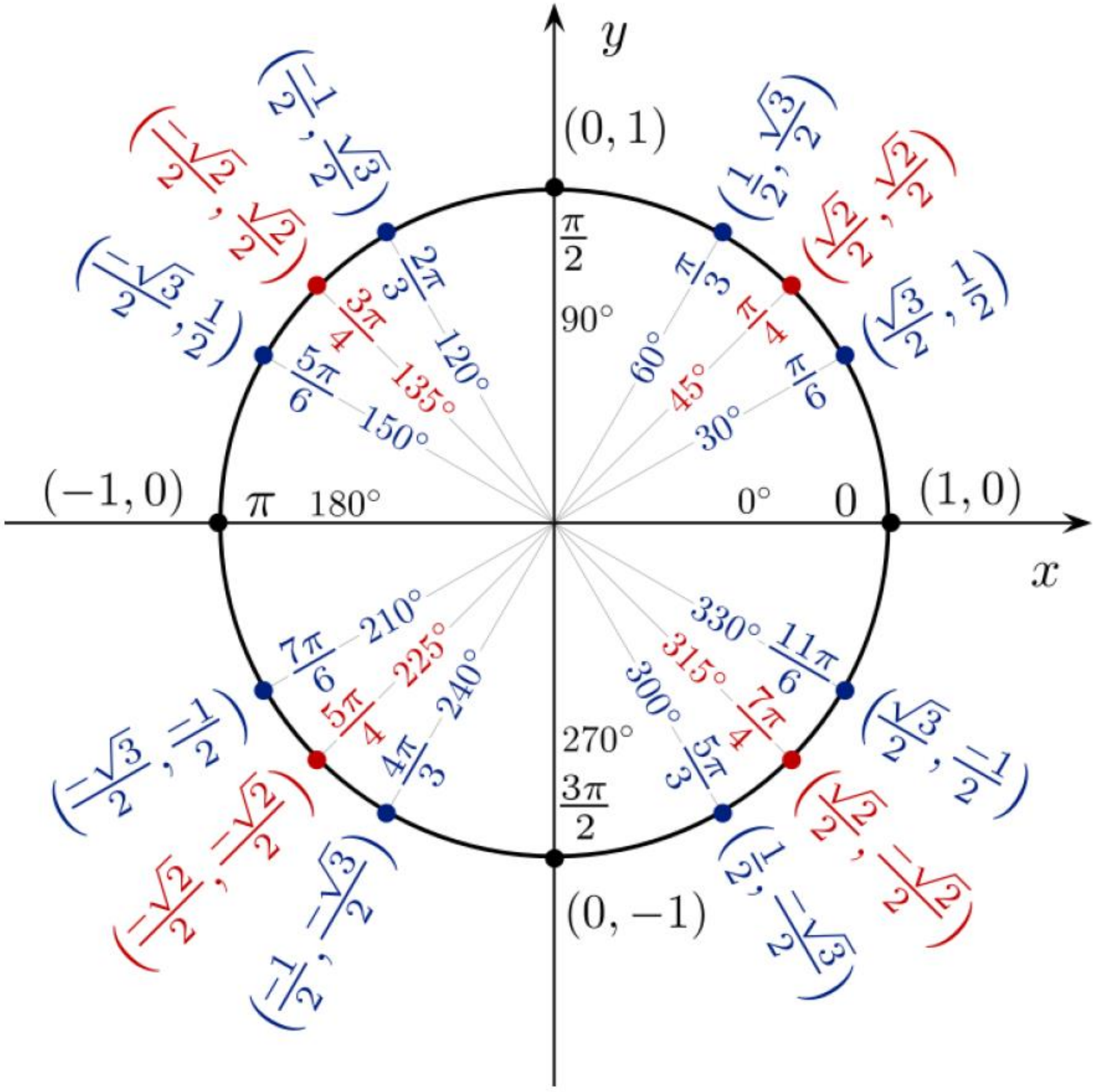
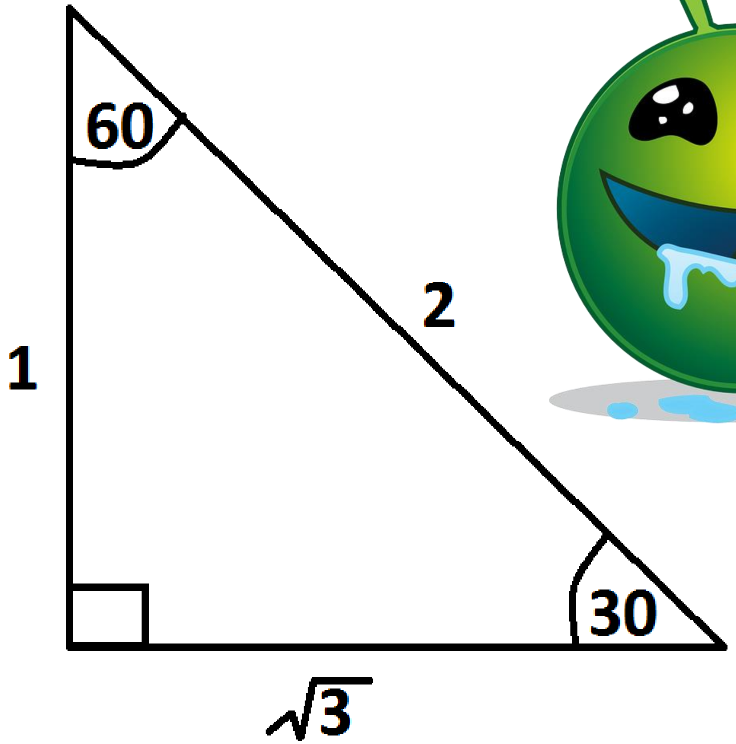
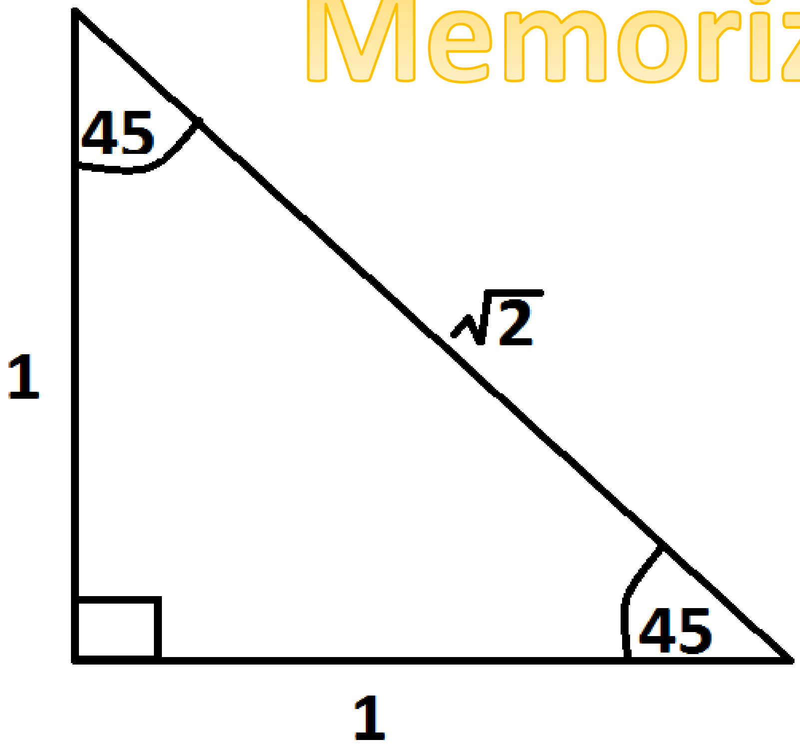
UNIT 3
ANALYTIC TRIGONOMETRY



y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
0
0°
π/6
30°
π/4
45°
π/3
60°
π/2
90°
2π/3
120°
3π/4
135°
5π/6
150°
π
180°
7π/6
210°
5π/4
225°
4π/3
240°
3π/2
270°
5π/3
300°
7π/4
315°
11π/6
330°
(√3/2, 1/2)
(√2/2, √2/2)
(1/2, √3/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

If you are given a problem that has an angle measurement of 45^{o}…30^{o}…or 60^{o}
…then you are in luck…!!!
These angle measurements belong to special triangles.





Memorize!!!

…so…how do you use these special right-angled triangles to find trig ratios…?
If the θ value you are given has one of these angles…then its easy…

Memorize!!!

Basic Reciprocal and Quotient Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

I WILL NOT GIVE THESE FORMULAE IN A TEST OR EXAM!!!
MEMORIZE IT!!!

Sum and Difference Identities

$\sin(\alpha+\beta) = \sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta$	$\sin(\alpha-\beta) = \sin\alpha\,\cos\beta - \cos\alpha\,\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta$	$\cos(\alpha-\beta) = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta$
$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Memorize!!!

Double Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
$\cos 2\theta = 2\cos^2\theta - 1$	$\cos 2\theta = 1 - 2\sin^2\theta$	

Half-Angle Identities

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$	$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$
$\tan\dfrac{\theta}{2} = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$	

Sum and difference identities

What is an identity?

An identity is a mathematical statement that uses an equal sign…=…and is always true for all values in the domain of either side of the equal sign.

Showing that the statement is correct for **SOME** values is **NOT ENOUGH**.

The process of showing that the statement is always true is called **PROOF!**

Sine & Cosine - Sum/Difference Identities

$\sin(\alpha+\beta) = sin\alpha\ cos\beta + cos\alpha\ sin\beta$	$\sin(\alpha-\beta) = sin\alpha\ cos\beta - cos\alpha\ sin\beta$
$\cos(\alpha+\beta) = cos\alpha\ cos\beta - sin\alpha\ sin\beta$	$\cos(\alpha-\beta) = cos\alpha\ cos\beta + sin\alpha\ sin\beta$

Tangent - Sum/Difference Identities

$\tan(\alpha+\beta) = \frac{tan\ \alpha + tan\ \beta}{1 - tan\ \alpha\ tan\ \beta}$	$\tan(\alpha-\beta) = \frac{tan\ \alpha - tan\ \beta}{1 + tan\ \alpha\ tan\ \beta}$

Example 1

Find the exact value of $sin75°$

$$sin75^o$$
$$= \sin(30^o + 45^o)$$
$$= sin30\ cos45 + cos30\ sin45$$
$$= \frac{1}{2}.\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

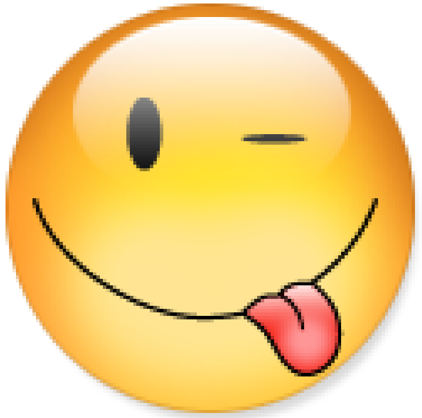
Example 2

Find the exact value of $cos75^o$

$$cos75^o$$
$$= \cos(30 + 45)$$
$$= cos30. \cos 45 - \sin 30. \sin 45$$
$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Solving equations using identities

Solving an equation means to determine specific values that satisfy the equation…and an equation may not be an identity. The number of solutions depends on the restriction of the domain.



For example: Solve the equation

$$\frac{tan14x - tan13x}{1 + tan14x \times tan13x} = \frac{\sqrt{3}}{3}$$

$$\tan(14x - 13x) = \frac{\sqrt{3}}{3} \ldots (\text{tan difference identity})$$

$$tanx = \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}\ and \frac{7\pi}{6}\ for\ 0 \leq x < 2\pi$$

For $x \in R$…there are limited number of solutions…and we will write a general solution:

$x = \frac{\pi}{6} \pm k\pi \ldots k \in Z\ and\ \pi$ is the period of $tanx$

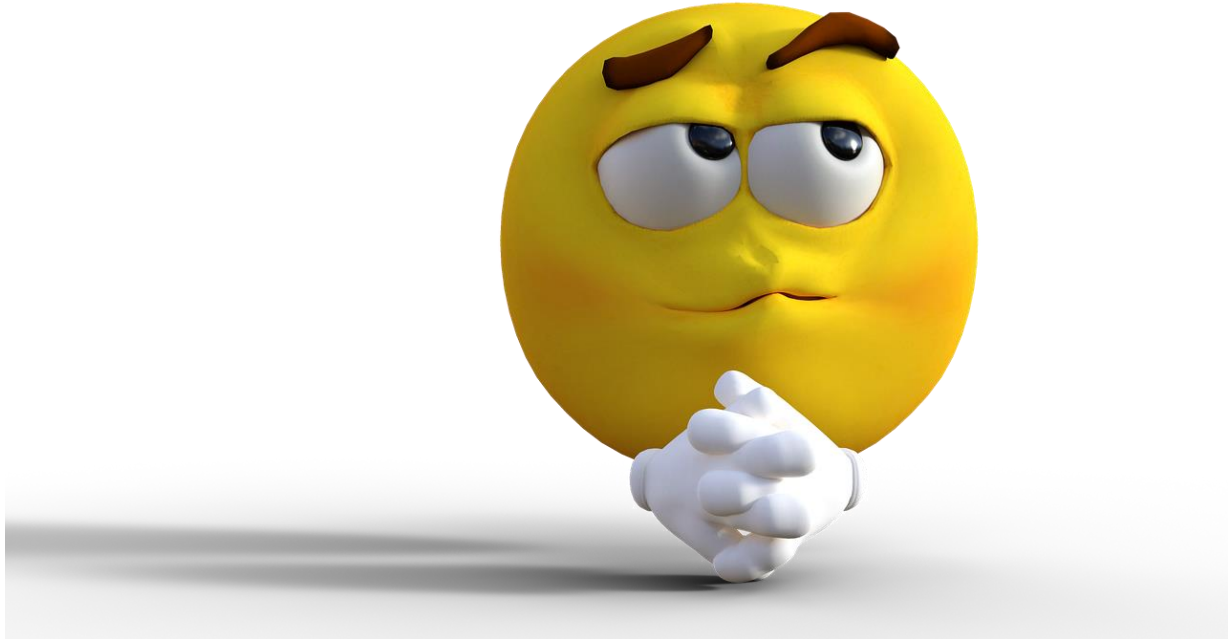
SOME EXAMPLES

$$\frac{1}{sinx} + \frac{1}{cosx}$$

$$(sinx + 2)(sinx - 5)$$

$$\frac{1}{sinx}+\frac{1}{cosx}$$
$$=\frac{1}{sinx}.\frac{cosx}{cosx}+\frac{1}{cosx}.\frac{sinx}{sinx}$$
$$=\frac{cosx}{sinxcosx}+\frac{sinx}{cosxsinx}$$
$$=\frac{cosx+sinx}{sinxcosx}$$

$$(sinx+2)(sinx-5)$$
$$=sinxsinx-5sinx+2sinx-10$$
$$=\sin^2 x-3sinx-10$$



Verify the identity: $sinx\ cotx = cosx$

Prove: $tanx + cosx = sinx(secx + cotx)$

Prove: $\frac{\cos^4 x - \sin^4 x}{\cos^2 x} = 1 - \tan^2 x$

Prove: $1 + cosx = \frac{\sin^2 x}{1 - cosx}$

Prove: $tanx + cotx = secx\, .\, cscx$

Verify the identity: $sinx\ cotx = cosx$

$$sinx\ cotx = sinx\ .\frac{cosx}{sinx}$$
$$= \frac{sinx\ cosx}{sinx}$$
$$= cosx$$

Prove: $tanx + cosx = sinx(secx + cotx)$

$$sinx(secx + cotx)$$
$$= sinx.\frac{1}{cosx} + sinx.\frac{cosx}{sinx}$$
$$= \frac{sinx}{cosx} + cosx$$
$$= tanx + cosx$$

Prove: $\frac{\cos^4 x - \sin^4 x}{\cos^2 x} = 1 - \tan^2 x$

Pythagorean identity

$$\frac{\cos^4 x - \sin^4 x}{\cos^2 x}$$

$$= \frac{(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)}{\cos^2 x}$$

$$= \frac{1(\cos^2 x - \sin^2 x)}{\cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 - \tan^2 x$$

Prove: $1 + cosx = \frac{\sin^2 x}{1-cosx}$

$$\frac{\sin^2 x}{1 - cosx}$$

$$= \frac{1 - \cos^2 x}{1 - \cos x}$$

$$= \frac{(1 - cosx)(1 + cosx)}{1 - cosx}$$

$$1 + cosx$$

Prove: $tanx + cotx = secx\,.cscx$

$$tanx + cotx$$

$$= \frac{sinx}{cosx}.\frac{sinx}{sinx} + \frac{cosx}{sinx}.\frac{cosx}{cosx}$$

$$= \frac{\sin^2 x + \cos^2 x}{cosx.sinx}$$

$$= \frac{1}{cosx\ sinx}$$

$$= \frac{1}{cosx}.\frac{1}{sinx}$$

$$= secx.cscx$$

Prove: $\frac{sinx}{1+cosx}+\frac{1+cosx}{sinx}=2cscx$

Prove: $\frac{1+sinx}{cosx}=\frac{cosx}{1-sinx}$

Prove: $\frac{1+tanx}{1+cotx}=tanx$

Prove: $\frac{sinx}{1+cosx}+\frac{1+cosx}{sinx}=2cscx$

$$\frac{sinx}{1+cosx}+\frac{1+cosx}{sinx}$$
$$=\frac{sinx}{sinx}.\frac{sinx}{1+cosx}+\frac{1+cosx}{sinx}.\frac{1+cosx}{1+cosx}$$
$$=\frac{\sin^2 x+(1+cosx)^2}{sinx(1+cosx)}$$
$$=\frac{\sin^2 x+1+2cosx+\cos^2 x}{sinx(1+cosx)}$$
$$=\frac{2+2cosx}{sinx(1+cosx)}$$
$$=\frac{2(1+cosx)}{sinx(1+cosx)}$$
$$=\frac{2}{sinx}$$
$$=2cscx$$

Prove: $\frac{1+sinx}{cosx} = \frac{cosx}{1-sinx}$

$$\frac{cosx}{1-sinx}$$

$$= \frac{cosx}{1-sinx} \cdot \frac{1+sinx}{1+sinx}$$

$$= \frac{cosx(1+sinx)}{1-\sin^2 x}$$

$$= \frac{cosx(1+sinx)}{\cos^2 x}$$

$$= \frac{1+sinx}{cosx}$$

Prove: $\frac{1+tanx}{1+cotx} = tanx$

$$\frac{1 + tanx}{1 + cotx}$$

$$= \frac{1 + \frac{sinx}{cosx}}{1 + \frac{cosx}{sinx}}$$

$$= \frac{\frac{cosx + sinx}{cosx}}{\frac{sinx + cosx}{sinx}}$$

$$= \frac{cosx + sinx}{cosx} \cdot \frac{sinx}{sinx + cosx}$$

$$= \frac{sinx}{cosx} = tanx$$

Verify the following identities:

$a)\, sinx.\, secx = tanx$

$b)\, \sin^2 x = tanx\ cotx\ - \cos^2 x$

$c)\, sinx(cotx + tanx) = secx$

$d)\, \sec^2 x\ csc^2 x = \sec^2 x + \csc^2 x$

$$\boldsymbol{sinx.secx = tanx}$$

$$sinx.\frac{1}{cosx}$$

$$= \frac{sinx}{cosx} = tanx$$

$$\mathbf{sin^2}\,\boldsymbol{x} = \boldsymbol{tanx\ cotx}\ - \mathbf{cos^2}\,\boldsymbol{x}$$

$$tanx\ cotx\ - \cos^2 x$$

$$= tanx.\frac{1}{tanx} - \cos^2 x$$

$$= 1 - \cos^2 x$$

$$= \sin^2 x$$

$$\boldsymbol{sinx(cotx + tanx) = secx}$$

$$
\begin{aligned}
&sinx(cotx + tanx)\\
&= sinx\left(\frac{cosx}{sinx} + \frac{sinx}{cosx}\right)\\
&= sinx\left(\frac{cosx.cosx}{sinx.cosx} + \frac{sinx.sinx}{cosx.sinx}\right)\\
&= sinx\left(\frac{\cos^2 x + \sin^2 x}{sinx.cosx}\right)\\
&= \frac{sinx(1)}{sinx.cosx}\\
&= \frac{1}{cosx} = secx
\end{aligned}
$$

$$\mathbf{sec^2\,x\,csc^2x = sec^2\,x + csc^2\,x}$$

$$\sec^2 x + \csc^2 x$$
$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$
$$= \frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\sin^2 x cos^2 x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x cos^2 x}$$
$$= \frac{1}{\sin^2 x cos^2 x}$$
$$= \frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$$
$$= \csc^2 x . \sec^2 x$$

$a)\ \sec x \cot x = \csc x$

$b)\ sinxtanx + cosx = secx$

$c)\ \cos x - \cos x sin^2 x = \cos^3 x$

$d)\ \frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2\sec x$

$$\mathbf{sec}\,\boldsymbol{x}\boldsymbol{cot}\ \boldsymbol{x} = \mathbf{csc}\,\boldsymbol{x}$$

$$\sec x cot\ x = \frac{1}{\cos x} . \frac{\cos x}{\sin x}$$

$$= \frac{1}{\sin x} = cscx$$

$$\boldsymbol{sinxtanx} + \boldsymbol{cosx} = \boldsymbol{secx}$$

$$\sin x \tan x + \cos x$$

$$= \sin x \left(\frac{\sin x}{\cos x}\right) + \cos x$$

$$= \frac{\sin^2 x}{cosx} + cosx$$

$$= \frac{\sin^2 x}{cosx} + \frac{cosx}{cosx} . cosx$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

$$\boldsymbol{\cos x - \cos x sin^2 x = \cos^3 x}$$

$$\cos x - \cos x sin^2 x$$
$$= \cos x(1 - \sin^2 x)$$
$$= \cos x . \cos^2 x$$
$$= \cos^3 x$$

$$\boldsymbol{\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}=2\sec x}$$

$$\begin{aligned}
&\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}\\
&=\frac{\cos x(\cos x)}{(1+\sin x)(\cos x)}+\frac{(1+\sin x)(1+\sin x)}{(1+\sin x)(\cos x)}\\
&=\frac{\cos^2 x}{(1+\sin x)(\cos x)}+\frac{1+2\sin x+\sin^2 x}{(1+\sin x)(\cos x)}\\
&=\frac{\left(\sin^2 x+\cos^2 x\right)+1+2\sin x}{(1+\sin x)(\cos x)}\\
&=\frac{1+1+2\sin x}{(1+\sin x)(\cos x)}\\
&=\frac{2+2\sin x}{(1+\sin x)(\cos x)}\\
&=\frac{2(1+\sin x)}{(1+\sin x)(\cos x)}\\
&=\frac{2}{\cos x}=2\sec x
\end{aligned}$$

Prove the following identities:

1. $\dfrac{\sin(2x)}{2\sin^2 x} = \cot x$
2. $\sin^2 x - \cos^2 x = \dfrac{\tan x - \cot x}{\tan x + \cot x}$
3. $\dfrac{\sin^2 x}{1+\cos x} = 1 - \dfrac{1}{\sec x}$
4. $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$
5. $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = 2\sec^2 x$

$$\frac{\sin(2x)}{2\sin^2 x} = \cot x$$

$$\frac{\sin(2x)}{2\sin^2 x}$$
$$= \frac{2\sin x cos\ x}{2\sin x sin\ x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$

$$\sin^2 x - \cos^2 x = \frac{\tan x - \cot x}{\tan x + \cot x}$$

$$\frac{\tan x - \cot x}{\tan x + \cot x}$$
$$= \frac{\frac{sinx}{cosx} - \frac{cosx}{sinx}}{\frac{sinx}{cosx} + \frac{cosx}{sinx}}$$
$$= \frac{\frac{\sin^2 x - \cos^2 x}{\sin x cos\ x}}{\frac{\sin^2 x + \cos^2 x}{\sin x cos\ x}}$$
$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x}$$
$$= \sin^2 x - \cos^2 x$$

$$\frac{\sin^2 x}{1+\cos x} = 1 - \frac{1}{\sec x}$$

$$1 - \frac{1}{\sec x}$$

$$= (1-\cos x).\frac{1+\cos x}{1+\cos x}$$

$$= \frac{(1-\cos x)(1+\cos x)}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1+\cos^2 x}$$

$$= \frac{\sin^2 x}{1+cosx}$$

$$\cos^4 x - \sin^4 x = 2\cos^2 x - 1$$

$$\begin{aligned}
&\cos^4 x - \sin^4 x \\
&= \left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right) \\
&= 1(\cos^2 x - \sin^2 x) \\
&= \cos^2 x - (1 - \cos^2 x) \\
&= \cos^2 x - 1 + \cos^2 x \\
&= 2\cos^2 x - 1
\end{aligned}$$

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x}$$

$$\frac{1 + sinx}{1 + sinx} . \frac{1}{1 - sinx} + \frac{1 - sinx}{1 - sinx} . \frac{1}{1 + sinx}$$

$$= \frac{1 + sinx + 1 - sinx}{1 + sinx . 1 - sinx}$$

$$= \frac{2}{1 - \sin^2 x}$$

$$= \frac{2}{\cos^2 x} = 2\sec^2 x$$

Double angle identities

Double Angle Identities

$\sin 2\theta = 2sin\ \theta\ cos\ \theta$	$\cos 2\theta = cos^2\theta - sin^2\theta$	$\tan 2\theta = \dfrac{2\tan\theta}{1 - tan^2\theta}$
$\cos 2\theta = 2cos^2\theta - 1$	$\cos 2\theta = 1 - 2sin^2\theta$	

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

Half angle identities

Half-Angle Identities

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$	$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$
$\tan\frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$	

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Reciprocal and quotient identities

Reciprocal Identities

$\sin\theta = \frac{1}{csc\ \theta}, \sin\theta \neq 0$	$\cos\theta = \frac{1}{sec\ \theta}, \cos\theta \neq 0$	$\tan\theta = \frac{1}{cot\ \theta}, \sin\theta \neq 0$

Quotient Identities

$\tan\theta = \frac{1}{cot\ \theta} = \frac{sin\ \theta}{cos\ \theta}, \cos\theta \neq 0$	$\cot\theta = \frac{1}{tan\ \theta} = \frac{cos\ \theta}{sin\ \theta}, \sin\theta \neq 0$

Solving equations with double angles

For a trigonometric equation with $sinbx$ … $cosbx$ … $tanbx$…keep in mind that the periods of the periodic functions are changed by the b. This will result in horizontal compression or expansion of the graph…which will directly affect the number of zeros in the function.

For example…find the answers of the equation $tan2x = -1$ in the domain

$0 \leq x < 2\pi$

In this question…the $2x$ is the new θ. So lets determine the general solutions first. Then we will use the general solution to determine all possible solutions in the domain.

$$2x = \tan^{-1}(-1)$$

$$2x = \frac{3\pi}{4} \ldots or\ 2x = \frac{7\pi}{4}$$

$$x = \frac{3\pi}{8} \ldots or\ x = \frac{7\pi}{8}$$

The general solutions are $x = \frac{3\pi}{8} \pm \frac{\pi}{2}k \ldots or \ldots x = \frac{7\pi}{8} \pm \frac{\pi}{2}k \ldots k \in Z$…and $\frac{\pi}{2}$ is the period of $tan2x$

Choose different values of k to find all solutions in the domain.

For $k = -1 \ldots 0 \ldots 1 \ldots 2 \ldots we\ get\ \frac{3\pi}{8} \ldots \frac{7\pi}{8} \ldots \frac{11\pi}{8} \ldots \frac{15\pi}{8}$

SOME EXAMPLES

1. When proving an identity…why cant we move an expression from one side of the equation to the other side?

Proving in mathematics, is about evaluating 1 side of the expression so that the end result equals to the opposite side. Moving an expression from 1 side to the other would not logically and mathematically prove this.

Verify: $(sinx + cosx)^2 = 1 + sin2x$

$$(sinx + cosx)^2$$

$$= (sinx + cosx)(sinx + cosx)$$
$$= \sin^2 x + sinxcosx + sinxcosx + \cos^2 x$$
$$= \sin^2 x + 2sinxcosx + \cos^2 x$$
$$= \sin^2 x + sin2x + \cos^2 x$$
$$= \sin^2 x + \cos^2 x + sin2x$$
$$= 1 + sin2x$$

More Random examples.

$$\sin x\left(\csc x + \sin x \sec^2 x\right) = \sec^2 x$$

$$2\cos^2 x + \sin^2 x = \cos^2 x + 1$$

$$2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\left(\sin x + \cos x\right)^2 + \left(\sin x - \cos x\right)^2 = 2$$

$$\frac{\cot x}{\csc x} = \cos x$$

$$\tan x + \cot x = \sec x \csc x$$

$$\sin x\left(\csc x + \sin x \sec^2 x\right) = \sec^2 x$$

$$\boxed{\begin{array}{l} \sin x \csc x + \sin^2 x \sec^2 x \\ 1 + \dfrac{\sin^2 x}{\cos^2 x} \\ 1 + \tan^2 x \\ \sec^2 x \end{array}}$$

$$2\cos^2 x + \sin^2 x = \cos^2 x + 1$$

$$\boxed{\begin{array}{l} \cos^2 x + \cos^2 x + \sin^2 x \\ \cos^2 x + 1 \end{array}}$$

$$2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\boxed{\begin{array}{l} 2\left(1 - \sin^2 x\right) - 1 \\ 2 - 2\sin^2 x - 1 \\ 1 - 2\sin^2 x \end{array}}$$

$$\left(\sin x + \cos x\right)^2 + \left(\sin x - \cos x\right)^2 = 2$$

$$\boxed{\begin{array}{l} \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x \\ \sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x \\ 2 \end{array}}$$

$$\frac{\cot x}{\csc x} = \cos x$$

$$\boxed{\begin{array}{l} \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} \\ \cos x \end{array}}$$

$$\tan x + \cot x = \sec x \csc x$$

$$\boxed{\begin{array}{l} \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} \\ \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\ \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x} \\ \sec x \csc x \end{array}}$$

More Random examples.

$$\sec x - \cos x = \sin x \tan x$$

$$\sin x + \cos x \cot x = \csc x$$

$$\frac{\cot x + 1}{\cot x - 1} = \frac{1 + \tan x}{1 - \tan x}$$

$$\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$$

$$\frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$$

$$\sec x - \cos x = \sin x \tan x$$

$$\frac{1}{\cos x} - \cos x$$

$$\frac{1-\cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} = \sin x \tan x$$

$$\frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$$

$$\left(\frac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x}\right)\left(\frac{\sin x \cos x}{\sin x \cos x}\right)$$

$$\frac{\cos x + \cos^2 x}{\sin^2 x + \sin^2 x \cos x} = \frac{\cos x(1+\cos x)}{\sin^2 x(1+\cos x)}$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = \cot x \csc x$$

$$\sin x + \cos x \cot x = \csc x$$

$$\sin x + \frac{\cos^2 x}{\sin x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$\frac{1}{\sin x} = \csc x$$

$$\frac{\cot x + 1}{\cot x - 1} = \frac{1 + \tan x}{1 - \tan x}$$

$$\left(\frac{\frac{1}{\tan x} + 1}{\frac{1}{\tan x} - 1}\right)\left(\frac{\tan x}{\tan x}\right)$$

$$\frac{1 + \tan x}{1 - \tan x}$$

$$\frac{\sec^2 x - 1}{\sec^2 x} = \sin^2 x$$

$$1 - \frac{1}{\sec^2 x}$$

$$1 - \cos^2 x$$

$$\sin^2 x$$

Prove:

$$\cot x + \tan x = \csc x . \sec x$$

Prove:

$$\csc x . \cos^2 x + \sin x = \csc x$$

Prove:

$$\frac{\cot x}{\csc x} = \cos x$$

Prove:

$$\cos^2 x = \frac{\cot x . \sin x}{\sec x}$$

Prove:

$$\frac{\sin(x - y)}{\sin(x + y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$$

Prove:

$$\cot x + \tan x = \csc x . \sec x$$

$$\cot x + \tan x$$

$$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$= \frac{\cos x . \cos x}{\sin x . \cos x} + \frac{\sin x . \sin x}{\cos x . \sin x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x . \cos x}$$

$$= \frac{1}{\sin x . \cos x}$$

$$= \frac{1}{\sin x} . \frac{1}{\cos x}$$

$$= \csc x . \sec x$$

Prove:

$$\csc x . \cos^2 x + \sin x = \csc x$$

$$\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1} \ldots\ldots\ldots \times \frac{\sin x}{sinx}$$

$$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

Prove:

$$\frac{\cot x}{\csc x} = \cos x$$

$$\frac{\cot x}{\csc x}$$

$$= \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$= \frac{\cos x}{\sin x} . \sin \text{x}$$

$$= \cos x$$

Prove:

$$\cos^2 x = \frac{\cot x . \sin x}{\sec x}$$

$$= \frac{\frac{\cos x}{\sin x} . \frac{\sin x}{1}}{\frac{1}{\cos x}}$$

$$= \frac{\cos x}{\frac{1}{\cos x}}$$

$$= \cos x \times \cos x$$

$$= \cos^2 x$$

Prove:

$$\frac{\sin(x-y)}{\sin(x+y)} = \frac{\tan x - \tan y}{\tan x + \tan y}$$

$$\frac{\sin(x-y)}{\sin(x+y)}$$

$$= \frac{\sin x.\cos y - \cos x.\sin y}{\sin x.\cos y + \cos x.\sin y}$$

$$= \frac{\dfrac{\sin x.\cos y}{\cos x.\cos y} - \dfrac{\cos x.\sin y}{\cos x.\cos y}}{\dfrac{\sin x.\cos y}{\cos x.\cos y} + \dfrac{\cos x.\sin y}{\cos x.\cos y}}$$

$$= \frac{\dfrac{\sin x}{\cos x} - \dfrac{\sin y}{\cos y}}{\dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}} = \frac{\tan x - \tan y}{\tan x + \tan y}$$

Prove:

$$\sin(\pi - x) = \sin x$$

Prove:

$$\sin(x + \pi) = -\sin x$$

Prove:

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

Prove:

$$csc2x = \frac{1}{2}\csc x.\sec x$$

Evaluate:

$$\sin(15)$$

Evaluate:

$$\cot(\frac{5\pi}{12})$$

Prove:

$$\sin(\pi - x) = \sin x$$

$$\sin(\pi - x)$$
$$= \sin\pi.\cos x - \cos\pi.\sin x$$
$$= 0 - (-1).\sin x$$
$$= +1(\sin x)$$
$$= \sin x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos(\frac{\pi}{2} - x)$$
$$= \cos\frac{\pi}{2}.\cos x + \sin\frac{\pi}{2}.\sin x$$
$$= 0 + 1(\sin x)$$
$$= \sin x$$

Prove:

$$\sin(x+\pi) = -\sin x$$

$$\sin(x+\pi)$$
$$= \sin x . \cos \pi + \cos x . \sin \pi$$
$$= \sin x . (-1) + 0$$
$$= -\sin x$$

Prove:

$$csc2x = \frac{1}{2}\csc x.\sec x$$

$$\csc 2x$$

$$= \frac{1}{\sin 2x}$$

$$= \frac{1}{2\sin x cos\ x}$$

$$= \frac{1}{2\sin x} + \frac{1}{\cos x}$$

$$= \frac{1}{2}\csc x.\sec x$$

Evaluate:

$$\sin(15)$$
$$= \sin(45 - 30)$$
$$= \sin 45 . \cos 30 - \cos 45 . \sin 30$$
$$= \frac{\sqrt{2}}{2} . \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} . \frac{1}{2}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

Evaluate:

$$\cot(\frac{5\pi}{12})$$

$$= \tan(\frac{\pi}{2} - x)$$

$$= \tan(\frac{\pi}{2} - \frac{5\pi}{12})$$

$$= \frac{\tan\left(\frac{\pi}{2}\right) - \tan(\frac{5\pi}{12})}{1 + \tan\left(\frac{\pi}{2}\right).\tan(\frac{5\pi}{12})}$$

$$= \frac{0 - 2 + \sqrt{3}}{1 + 0.(-2 + \sqrt{3})}$$

$$= -2 + \sqrt{3}$$

If $\sin x = \frac{4}{5} \ldots 0 < x < 90 \ldots find \sin 2x$

Does $\sin x . \cos x = \frac{1}{2} \sin 2x$?

Prove:

$$\csc 2x = \frac{1}{2}(\tan x + \cot x)$$

Prove:

$$2 \csc x \cos^2 \frac{x}{2} = \frac{\sin x}{1 - \cos x}$$

If $\sin x = \frac{4}{5} \ldots 0 < x < 90 \ldots find \sin 2x$

$$\sin 2x = 2\sin x cos\, x$$
$$\sin^2 x + \cos^2 x = 1$$
$$\cos x = \sqrt{1 - \sin^2 x}$$
$$= \sqrt{1 - \left(\frac{4}{5}\right)^2}$$
$$= \frac{3}{5}$$
$$\cos x > 0 \ldots x\ in\ quadrant\ I$$
$$\sin 2x = 2\sin x cos\, x$$
$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$
$$= \frac{24}{25}$$

Does $\sin x . \cos x = \frac{1}{2} \sin 2x$?

$$Yes.$$

$$\sin x . \cos x = \frac{2 \sin x cos\ x}{2}$$

$$= \frac{\sin 2x}{2}$$

$$= \frac{1}{2} \sin 2x$$

Prove:

$$\csc 2x = \frac{1}{2}(\tan x + \cot x)$$

$$\frac{1}{2}(\tan x + \cot x)$$

$$= \frac{1}{2}\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \frac{1}{2}\left(\frac{\sin^2 x + \cos^2 x}{\cos x . \sin x}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\cos x . \sin x}\right)$$

$$= \frac{1}{2\cos x sin\ x}$$

$$= \frac{1}{\sin 2x} = csc2x$$

$$\boxed{\times \frac{sin}{sinx} \ldots \times \frac{\cos x}{\cos x}}$$

Prove:

$$2\csc x\cos^2\frac{x}{2} = \frac{\sin x}{1-\cos x}$$

$$2\csc x\cos^2\frac{x}{2}$$

$$= 2cscx\left(\frac{1+\cos x}{2}\right)$$

$$= \frac{1+\cos x}{\sin x}$$

$$= \frac{1+\cos x}{\sin x}\cdot\frac{1-\cos x}{1-\cos x}$$

$$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin x}{1-\cos x}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$
$$cscx = 1/\sin x$$

Multiply numerator and denominator by conjugate of numerator

Using the formula for the cosine of the difference of 2 angles…find the exact value of $\cos(\frac{5\pi}{4} - \frac{\pi}{6})$

Find the exact value of $\cos 75$

Find the exact value of $\tan(\frac{\pi}{6} + \frac{\pi}{4})$

Given $sinx = \frac{3}{5}$ and x in quadrant II. Determine the values of $\sin\frac{x}{2} \ldots \cos\frac{x}{2} \ldots and \tan\frac{x}{2}$.

Using the formula for the cosine of the difference of 2 angles…find the exact value of $\cos(\frac{5\pi}{4} - \frac{\pi}{6})$

$$\cos(\frac{5\pi}{4} - \frac{\pi}{6})$$

$$= \cos\left(\frac{5\pi}{4}\right).\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{5\pi}{4}\right).\sin(\frac{\pi}{6})$$

$$= \left(-\frac{\sqrt{2}}{2}\right).\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right).\frac{1}{2}$$

$$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{-\sqrt{6} - \sqrt{2}}{4}$$

Find the exact value of $\cos 75$

$$75 = 45 + 30$$
$$\cos(45 + 30)$$
$$= \cos 45 . \cos 30 - \sin 45 . \sin 30$$
$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2} - \frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

Find the exact value of $\tan(\frac{\pi}{6}+\frac{\pi}{4})$

$$\tan(\frac{\pi}{6}+\frac{\pi}{4})$$

$$=\frac{\tan\left(\frac{\pi}{6}\right)+\tan(\frac{\pi}{4})}{1-(\tan(\frac{\pi}{6}))(\tan(\frac{\pi}{4}))}$$

$$=\frac{\frac{1}{\sqrt{3}}+1}{1-(\frac{1}{\sqrt{3}})(1)}$$

$$=\frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$=\frac{1+\sqrt{3}}{\sqrt{3}}.\frac{\sqrt{3}}{\sqrt{3}-1}=\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Given $sinx = \frac{3}{5}$ and x in quadrant II. Determine the values of $\sin\frac{x}{2} \ldots \cos\frac{x}{2} \ldots and \tan\frac{x}{2}$.

Since x is in quadrant II…we have:

$$\cos x = -\sqrt{1 - \sin^2 x} = -\frac{4}{5}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$$
$$= \sqrt{\frac{1 + \frac{4}{5}}{2}} = \frac{3\sqrt{10}}{10}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}$$
$$= \sqrt{\frac{1 - \frac{4}{5}}{2}} = -\frac{\sqrt{10}}{10}$$

$$\tan\frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$
$$= -3$$

Prove:

$$\sin^4 x = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

Given $\cos x = \frac{5}{13}, \ldots \frac{3\pi}{2} < 0 < 2\pi \ldots find \sin 2x \ldots \cos 2x \ldots \tan 2x$

Develop a formula for $\cot 2x$ in terms of x

Prove:

$$\sin^4 x = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

$$\sin^4 x = \left(\sin^2 x\right)^2$$

$$= \left(\frac{1 - \cos 2x}{2}\right)^2$$

$$= \frac{1}{4}(1 + \cos^2 2x - 2\cos 2x)$$

$$= \frac{1}{4}(1 + \left(\frac{1 + \cos 4x}{2}\right) - 2\cos 2x)$$

$$= \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

Given $\cos x = \frac{5}{13}, \ldots \frac{3\pi}{2} < 0 < 2\pi \ldots find \sin 2x \ldots \cos 2x \ldots \tan 2x$

The fact that x is in quadrant IV…implies that $\sin x = -\sqrt{1 - \cos^2 x} = -\frac{12}{13}$

So…

$$\sin 2x = 2 \sin x cos\ x = -\frac{120}{169}$$

$$\cos 2x = 2\cos^2 x - 1 = -\frac{119}{169}$$

$$\tan 2x = \frac{\sin x}{\cos x} = \frac{120}{119}$$

Develop a formula for $\cot 2x$ in terms of x

Using the formula for $\tan 2x$ … *we have* …

$$\cot 2x = \frac{1}{\tan 2x} = \frac{1-\tan^2 x}{2\tan x}$$
$$= \frac{1}{2}\left(\frac{1}{\tan x} - \tan x\right)$$
$$= \frac{1}{2}(\cot x - \tan x)$$

Prove the following:

- $\sec x - \tan x . sinx = \frac{1}{\sec x}$
- $\frac{1+\cos x}{\sin x} = \csc x + \cot x$
- $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$
- $\frac{\sec x . \sin x}{\tan x + \cot x} = \sin^2 x$

- $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$
- $\csc^2 x . \tan^2 x - 1 = \tan^2 x$
- $\frac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$
- $\tan^2 x . \sin^2 x = \tan^2 x - \sin^2 x$
- $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$
- $(\sin x + \cos x)(\tan \mathrm{x} + \cot \mathrm{x}) = \sec \mathrm{x} + \csc \mathrm{x}$
- $\frac{\sin x + \tan x}{1 + \sec x} = \sin x$
- $\frac{\tan x}{\sec x} + \frac{\cot x}{\csc x} = \sin x + \cos x$
- $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$

$$\sec x - \tan x . sinx = \frac{1}{\sec x}$$

$$\sec x - \tan x . sinx$$
$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} . \sin x$$
$$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$
$$= \frac{1 - \sin^2 x}{\cos x}$$
$$= \frac{\cos^2 x}{\cos x}$$
$$= \cos x = \frac{1}{\sec x}$$

$$\frac{1 + \cos x}{\sin x} = \csc x + \cot x$$

$$\frac{1 + \cos x}{\sin x}$$
$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$
$$= \csc x + \cot x$$

$$\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$$

$$= \frac{\frac{1}{\cos x}}{\cos x} - \frac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}}$$
$$= \frac{1}{\cos x} . \frac{1}{\cos x} - \frac{\sin x}{\cos x} . \frac{\sin x}{\cos x}$$
$$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$
$$= \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$$

$$\frac{\sec x . \sin x}{\tan x + \cot x} = \sin^2 x$$

$$\frac{\sec x . \sin x}{\tan x + \cot x}$$
$$= \frac{\frac{1}{\cos x} . sinx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$
$$= \frac{\frac{\sin x}{\cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x cos\ x}}$$
$$= \frac{sinx}{\cos x} . \frac{\sin x . \cos x}{1}$$
$$= \sin x . \sin x = \sin^2 x$$

$$\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$
$$= 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$

$$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$$
$$= \sin^2 x + 2\sin x cos\ x + \cos^2 x + \sin^2 x - 2\sin x cos\ x + \cos^2 x$$
$$= \sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x$$
$$= 1 + 1 = 2$$

$$\csc^2 x . \tan^2 x - 1 = \tan^2 x$$
$$= \tan^2 x + \cot^2 x . \tan^2 x - 1$$
$$= \tan^2 x + 1 - 1$$
$$= \tan^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

$$\frac{\sec^2 x}{\sec^2 x - 1} = \csc^2 x$$
$$= \frac{\sec^2 x}{\tan^2 x}$$
$$= \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$$
$$= \frac{1}{\cos^2 x} . \frac{\cos^2 x}{\sin^2 x}$$
$$= \frac{1}{\sin^2 x} = \csc^2 x$$

$$\tan^2 x . \sin^2 x = \tan^2 x - \sin^2 x$$
$$= (\sec^2 x - 1)(\sin^2 x)$$
$$= \sec^2 x . \sin^2 x - \sin^2 x$$
$$= \frac{1}{\cos^2 x} . \sin^2 x - \sin^2 x$$
$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$
$$= \tan^2 x - \sin^2 x$$

$$(\sin x + \cos x)(\tan \mathrm{x} + \cot \mathrm{x}) = \sec \mathrm{x} + \csc \mathrm{x}$$

$$= \sin x tan\ x + \sin x cot\ x + \cos x tan\ x + \cos x cot\ x$$

$$= \sin x.\frac{\sin x}{\cos x} + \sin x.\frac{\cos x}{\sin x} + \cos x.\frac{\sin x}{\cos x} + \cos x\frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x} + \cos x + \sin x + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\cos x} + \frac{1}{\sin x}$$

$$= \sec x + \csc x$$

$$\frac{\sin x + \tan x}{1 + \sec x} = \sin x$$

$$= \frac{sinx + \frac{\sin x}{\cos x}}{1 + \frac{1}{\cos x}}$$

$$= \frac{\frac{\sin x cos\ x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{1}{\cos x}}$$

$$= \frac{\frac{\sin x cos\ x + \sin x}{\cos x}}{\frac{1 + \cos x}{\cos x}}$$

$$= \frac{\sin x. \cos x + \sin x}{\cos x}.\frac{\cos x}{1 + \cos x}$$

$$= \frac{\sin x cos\ x + \sin x}{1 + \cos x}$$

$$= \frac{\sin x(\cos x + 1)}{1 + \cos x}$$

$$= \sin x$$

$$\frac{\tan x}{\sec x}+\frac{\cot x}{\csc x}=\sin x+\cos x$$

$$=\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}+\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$=\frac{\sin x}{\cos x}.\cos x+\frac{\cos x}{\sin x}.\sin x$$

$$=\sin x+\cos x$$

$$\frac{\cos x+1}{\sin^3 x}=\frac{\csc x}{1-\cos x}$$

$$=\frac{\cos x+1}{\sin x.\sin^2 x}$$

$$=\frac{\cos x+1}{\sin x(1-\cos^2 x\ )}$$

$$=\frac{1}{\sin x(1-\cos x)}$$

$$=\frac{1}{\sin x}.\frac{1}{1-\cos x}$$

$$=\frac{\csc x}{1-\cos x}$$

1. Find the exact value of each trigonometric expression

a) $\sin(45+30)$

b) $\cos(\frac{3\pi}{4}+\frac{5\pi}{6})$

c) $\tan(300-45)$

d) $\sin -15$

e) $\cos(\frac{5\pi}{12})$

f) $\tan(15)$

When you are done…use the time to go over your trigonometry from this unit. Please study it

For the test…there will be NO formulae given!!!
You need to memorise it!!!

2. If $\cos x = \frac{1}{4}$ and x is in quadrant IV…find the exact value of $\sin(x-\frac{\pi}{6})$

$$\sin(45+30)$$
$$= \sin 45 . \cos 30 + \cos 45 . \sin 30$$
$$= \frac{\sqrt{2}}{2} . \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} . \frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos(\frac{3\pi}{4} + \frac{5\pi}{6})$$
$$= \cos\frac{3\pi}{4} . \cos\frac{5\pi}{6} - \sin\frac{3\pi}{4} . \sin\frac{5\pi}{6}$$
$$= -\frac{\sqrt{2}}{2} . \frac{-\sqrt{3}}{2} - (\frac{\sqrt{2}}{2} . \frac{1}{2})$$
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\tan(300 - 45)$$

$$= \frac{\tan 300 - \tan 45}{1 + \tan 300 . \tan 45}$$

$$= \frac{-\sqrt{3} - 1}{1 + \left(-\sqrt{3}\right). 1}$$

$$= \frac{(-\sqrt{3} - 1)}{1 - \sqrt{3}} . \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})} \ldots conjugate$$

$$= \frac{-\sqrt{3} - 1 - 3 - \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - 3} = \frac{-4 - 2\sqrt{3}}{-2}$$

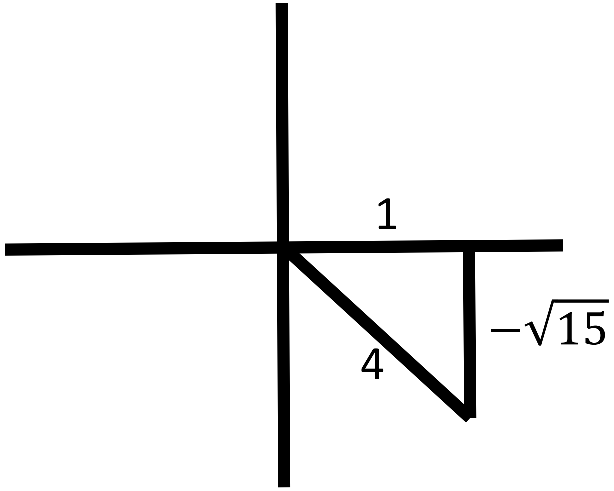
$$\sin -15$$
$$= \sin(30 - 45)$$
$$= \sin 30 . \cos 45 - \cos 30 . \sin 45$$
$$= \frac{1}{2} . \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} . \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cos(\frac{5\pi}{12})$$
$$= \cos(\frac{\pi}{6} + \frac{\pi}{4})$$
$$= \cos\frac{\pi}{6} . \cos\frac{\pi}{4} - \frac{sin\pi}{6} . \sin\frac{\pi}{4}$$
$$= \frac{\sqrt{3}}{2} . \frac{\sqrt{2}}{2} - \frac{1}{2} . \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan(15)$$
$$= \tan(45 - 30)$$

$$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$
$$= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \ldots Conjugate(3 - \sqrt{3})$$
$$= \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3\sqrt{3} + 3\sqrt{3} - 3}$$
$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

If $\cos x = \frac{1}{4}$ and x is in quadrant IV…find the exact value of $\sin(x - \frac{\pi}{6})$



$$\sin(x - \frac{\pi}{6})$$

$$= \sin x . \cos\frac{\pi}{6} - \sin\frac{\pi}{6} . \frac{1}{4}$$

$$= -\frac{\sqrt{15}}{4} . \frac{\sqrt{3}}{2} - \frac{1}{2} . \frac{1}{4}$$

$$= \frac{-3\sqrt{5}}{8} - \frac{1}{8}$$

$$= \frac{-3\sqrt{5} - 1}{8}$$