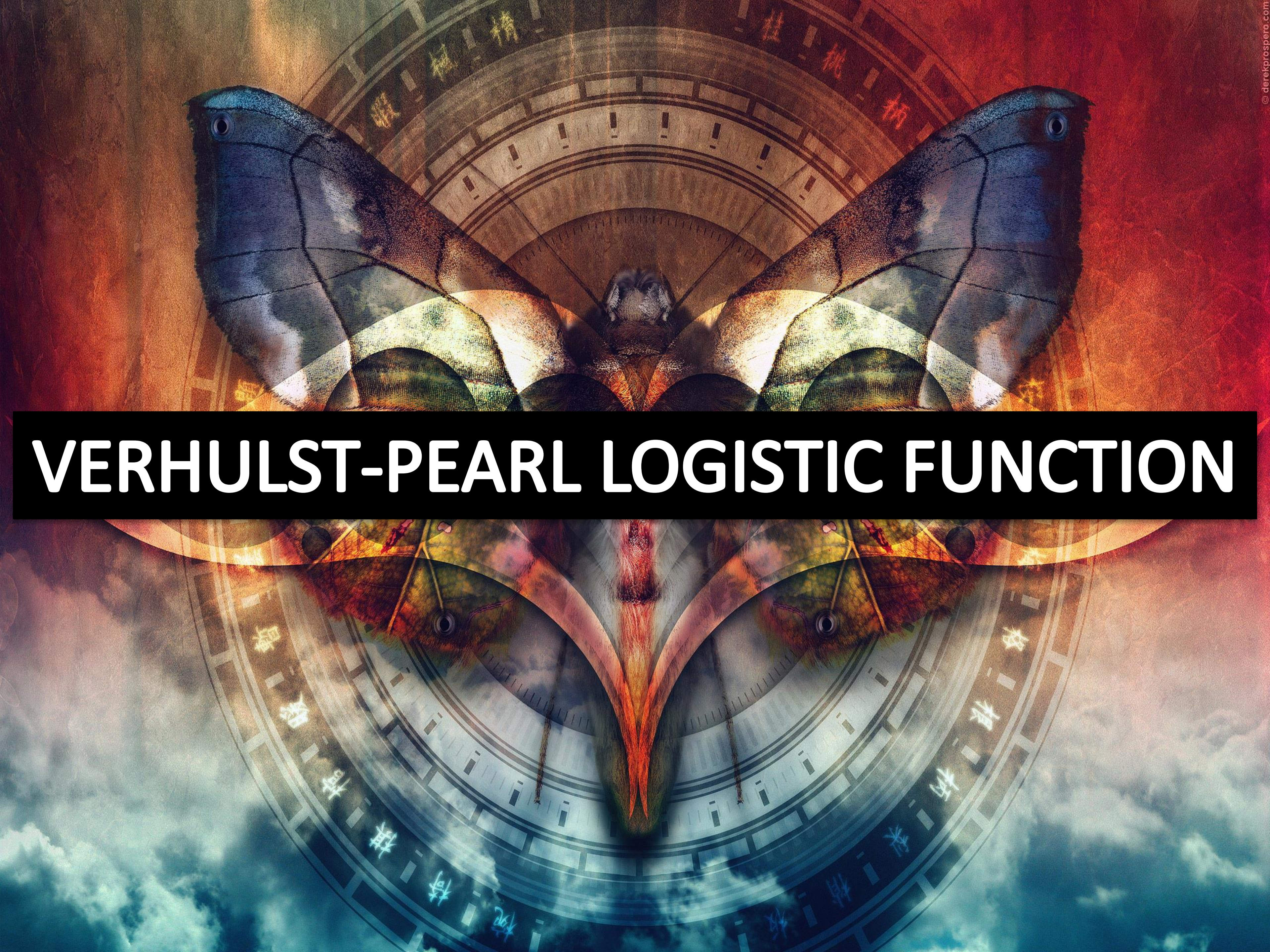
# VERHULST-PEARL LOGISTIC FUNCTION



Check This Guy Out…


…well his name is Pierre-Francois Verhulst

A good application of the logistic equation is a common model of population growth.

This was due to the work of Pierre-Francois Verhulst in 1838.

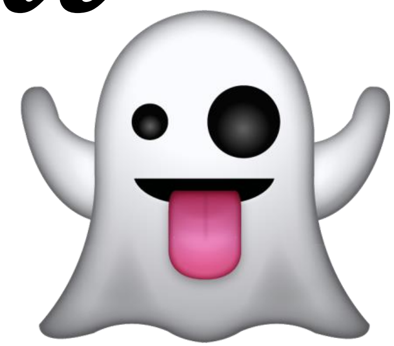
He stated that the rate of reproduction is proportional to both the existing population and the amount of available resources.

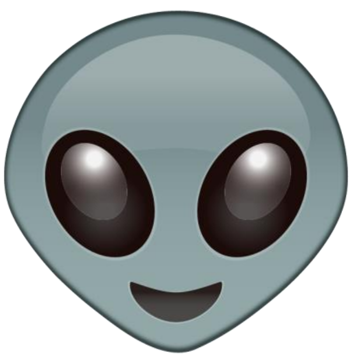
The equation is sometimes called the Verhulst-Pearl equation…following its rediscovery in 1920 by Raymond Pearl.

Members in a Population

Constant

$$\boldsymbol{N = N_0 \cdot e^{kt}}$$





Initial Value

Members in a Population

Carrying Capacity

$$N = \frac{M}{1 + be^{-ct}}$$

Logistic Growth Constant

**Example 1:**

Suppose the number of accounts in a new bank is to be a **maximum of 1600 accounts**.

**One year ago**…the initial number of accounts was **100**, and now there are **400** accounts.

If we know the number of opening accounts follows the logistic function…

**How many accounts will there be after 4 years from now?**

When t = 0…N = 100 &
M = 1600…

Here's what was given

$$N = \frac{M}{1 + be^{-ct}}$$

So here's our equation

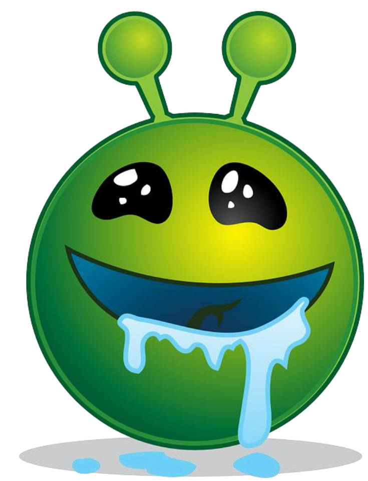
$$N = \frac{M}{1 + be^{-ct}}$$

$$100 = \frac{1600}{1 + b}$$

$$1 + b = 16$$

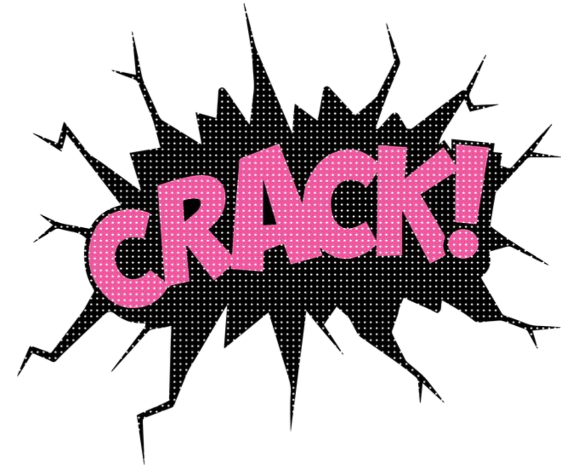
$$b = 15$$

Lets solve for b…



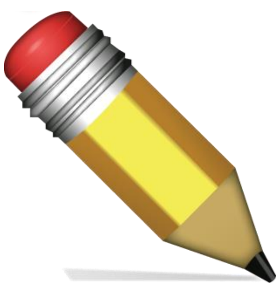
Now we have b

When t = 1…N = 400




$$400 = \frac{1600}{1 + 15e^{-c}}$$

$$c = \ln 5$$



After 4 years, t will be 5…

$$N = \frac{1600}{1 + 15\left(\frac{1}{5}\right)^{5}}$$

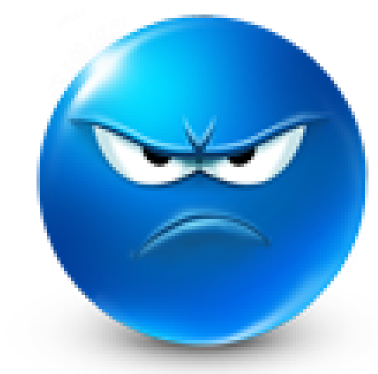
$$N = 1592$$

Here's our solution

**Example 2:**

In a community of 50 000 people, a survey is investigating the spread of a new flu.

When they started the survey, they found that 400 people knew of the flu.



After 1 week, 1250 people knew.

Find the number of people who knew 5 weeks after the survey began, assuming logistic growth.

$$M = 50\ 000$$
$$when\ t = 0$$
$$N = 400$$
$$t = 1\ \ \ldots N = 1250 \ldots$$

$$N = \frac{M}{1 + be^{-ct}}$$

$$400 = \frac{50000}{1 + b}$$

$$400(1 + b) = 50\ 000$$

$$400 + 400b = 50\ 000$$

$$b = \frac{50\ 000 - 400}{400}$$

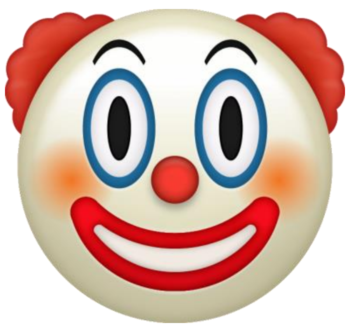
$$b = 124$$

$$1250 = \frac{50\ 000}{1 + 124e^{-c}}$$

$$50\ 000 = 1250\ (1 + 124.e^{-c})$$

$$50\ 000 = 1250 + 155\ 000.e^{-c}$$

$$e^{-c} = \frac{39}{124}$$



$$when\ t = 5$$

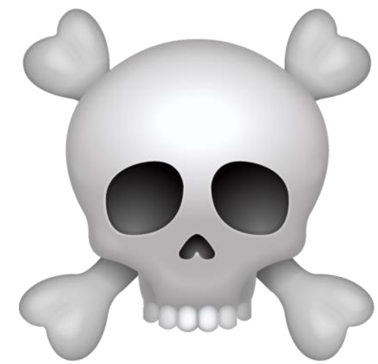
$$N = \frac{50\ 000}{1 + 124\left(\frac{39}{124}\right)^{5}}$$
$$= 36189$$

**Example 3:**

At t= 0, there is 1 person in a community of 1000 people who has the flu.

At most only 1000 people can have it.

Logistic growth constant c = 0.6030

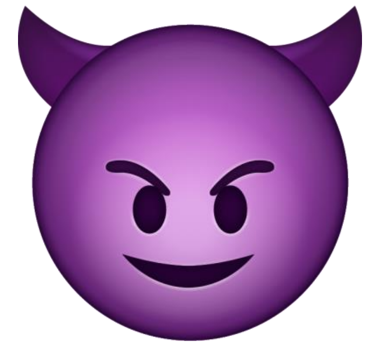


Estimate the number of people who will have the flu after 10 days.

$$N = \frac{M}{1 + be^{-ct}}$$

$$1 = \frac{1000}{1 + b}$$

$$1 + b = 1000$$
$$b = 999$$



$$N = \frac{1000}{1 + 999.\, e^{-(0.6030)(10)}}$$

$$N = 293.8$$

**Example 4:**

A population of rabbits in a meadow is observed to be 200 rabbits at time $t = 0$.

After a month, the rabbit population has increased by 4.

Using an initial population of 200 and a growth rate of 0.04, with a carrying capacity of 750 rabbits…

a) Predict the population after a year.

$$N = \frac{M}{1 + be^{-ct}}$$

$$200 = \frac{750}{1 + b.e^{-0.04(0)}}$$

$$200 = \frac{750}{1 + b}$$

$$200(1 + b) = 750$$

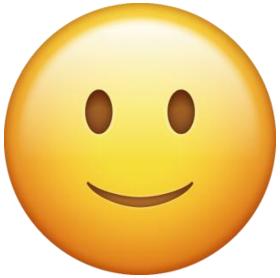
$$200 + 200b = 750$$

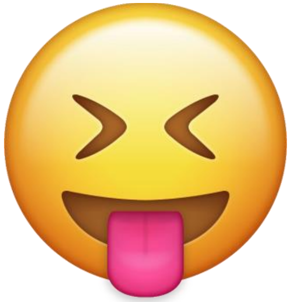
$$b = 2.75$$

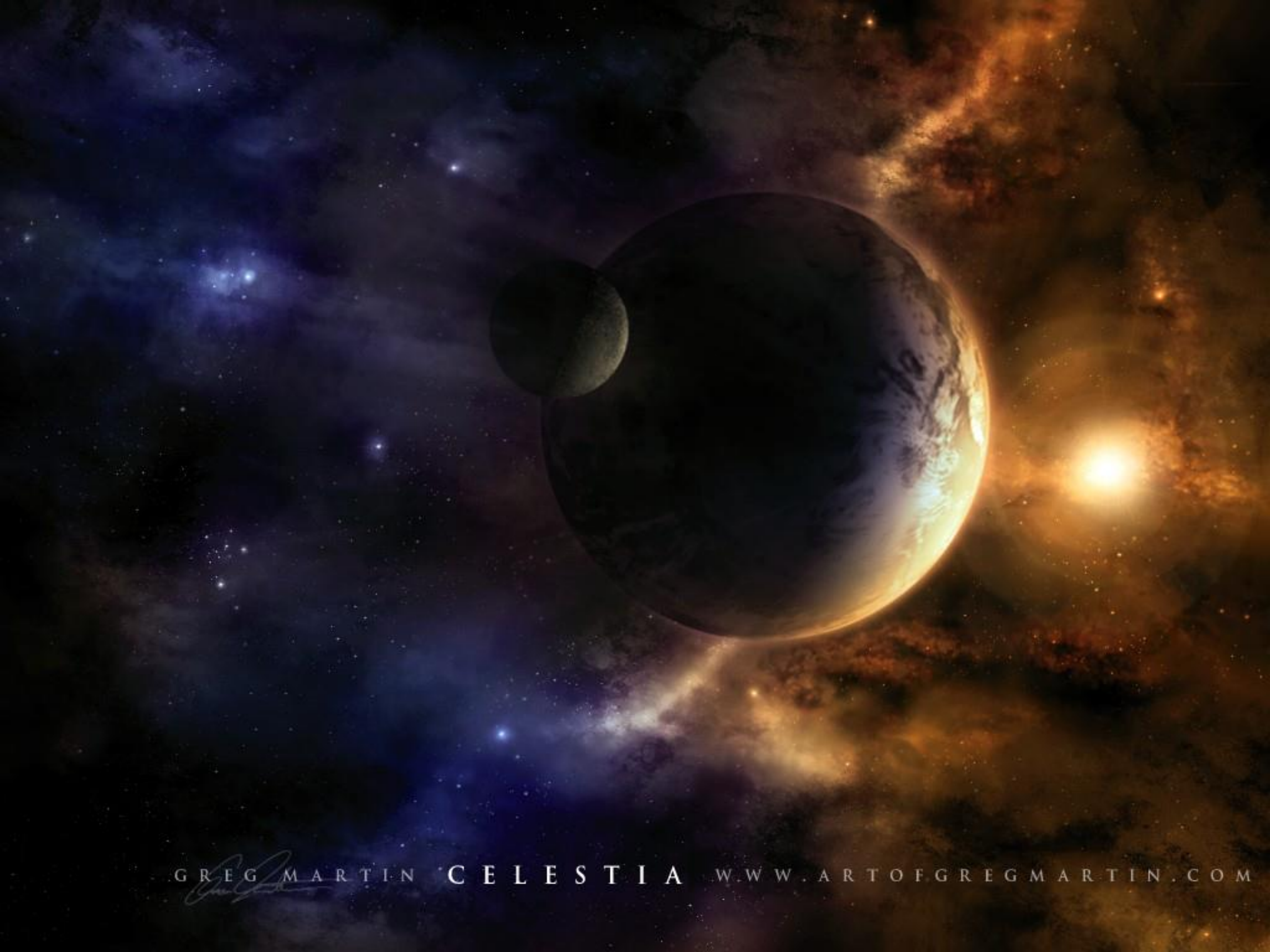
$$N = \frac{M}{1 + be^{-ct}}$$

$$= \frac{750}{1 + 2.75.e^{-0.04(12)}}$$

$$N = 277.6$$



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